VIBRATION AND BUCKLING OF COMPOSITE TWISTED PANELS SUBJECTED TO HYGROTHERMAL LOADING

A thesis submitted in partial fulfillment

of the requirements for the degree of

MASTER OF TECHNOLOGY

in

CIVIL ENGINEERING

(STRUCTURAL ENGINEERING)

BY

ASHISH SINGH

ROLL NO-211CE2031

DEPARTMENT OF CIVIL

NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

ROURKELA-769008, ODISHA, MAY 2013

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UNDER THE GUIDANCE OF

Prof. Shishir Kr. Sahu

DEPARTMENT OF CIVIL
NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA
ROURKELA-769008, ODISHA,
MAY 2013

DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA, ODISHA-769008

CERTIFICATE

This is to certify that the thesis entitled, **"VIBRATION AND BUCKLING OF COMPOSITE TWISTED PANELS SUBJECTED TO HYGROTHERMAL LOADING"** submitted by **ASHISH SINGH** bearing Roll no. **211CE2031** in partial fulfillment of the requirements for the award **of Master of Technology** degree in **Civil Engineering** with specialization in **"Structural Engineering"** during 2011-2013 session at the National Institute of Technology, Rourkela is an authentic work carried out by her under my supervision and guidance. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Place: Rourkela

Date: 31/05/2013

Prof. Shishir Kr. Sahu

Department of civil Engineering

National Institute of technology

Rourkela, Odisha-769008

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Ashish Singh

Roll no- 211ce2031

CONTENTS

ABSTRACT

The twisted composite cantilever have significant applications in aeronautical and aerospace industry civil, naval and other high-performance engineering applications due to their light weight, high-specific strength and stiffness, excellent thermal characteristics, ease in fabrication and other important specialties These structural members are often exposed to various service loads during their entire service life. The presence of temperature and moisture concentration in the environment may greatly reduce the stiffness and strength of the structures and may affect some design parameter such as vibration and stability characteristics of the structures. To avoid these typical problems caused by vibrations and stability, it is important to find out natural frequency, static stability of the composite laminated twisted cantilever panels under hygrothermal conditions. Therefore the vibration and stability behavior of laminated composite twisted panels subjected to hygrothermal conditions are studied in the present investigation.

A simple laminated model is developed for the vibration and stability analysis of laminated composite pre-twisted cantilever panel subjected to hygrothermal conditions. A computer program based on FEM in MATLAB environment is developed to perform all necessary computations. Here an eight noded isoparametric quadratic shell element with five degrees of freedom per node is used based on FSDT theory with hygrothermal loading. Element elastic stiffness matrices, mass matrices, geometric stiffness matrix due to hygrothermal loads and load vectors are derived using the principle of minimum potential energy. The influences of various parameters such as angle of twist, aspect ratio, ply-orientation, geometry and number of layers of laminate are studied on the vibration and buckling characteristics of laminated pre twisted cantilever panels for different temperatures and moisture concentrations. Numerical results are presented to show the effects of pre-twist angles, geometry and lamination details on the vibration and buckling characteristics of twisted plates. This can be used to the advantage of tailoring during design of composite twisted structures.

Keywords: Composite twisted panel, hygrothermal loading, stability

NOMENCLATURE

The principal symbols used in this thesis are presented for easy reference

English

a, b	Dimensions of the twisted panel
a/b	Aspect ratio of the twisted panel
Aij, Bij, Dij and Sij	Extensional, bending-stretching coupling, Bending and transverse shear stiffnesses
b/h	Width to thickness ratio of the twisted panel
[B]	Strain displacement matrix for the element
C, C0	elevated and reference moisture concentrations
[D]	Stress-strain
dx, dy	Element length in x and y-direction
dv	Volume of the element
E_1, E_2	Young's moduli of a lamina along and across the fibers, respectively
$G_{12}, G_{13,} G_{23}$	Shear moduli of a lamina with respect to 1, 2 and 3 axes
$[K_e]$	The elastic stiffness matrix
$[K_G^r]$	The initial stress stiffness matrix
$[K_{Ge}^a]$	The geometric stiffness matrix due to applied in-plane loads
k_x, k_y, k_{xy}	Curvatures of the plate
$(M_x,\ M_y\ and\ M_{xy})$	Internal moment resultants per unit length

M_x^N, M_y^N, M_{xy}^N	Initial internal moment resultants per unit length
M_x^i, M_y^i, M_{xy}^i	Non-mechanical moment resultants per unit length due to moisture and temperature
[N]	The shape function matrix
Ni	Shape function at a node i
Nx, Ny, Nxy	In-plane internal force resultants per unit length
N_x^i, N_y^i, N_{xy}^i	In-plane initial internal force resultants per unit length
N_x^N, N_y^N, N_{xy}^N	In-plane non-mechanical force resultants per unit length due to moisture and temperature
Nxx	Critical buckling load in x-direction
$\{P_e\}$	The element load vector due to external transverse static load
$\{P^N{}_e\}$	The element load vector due to hygrothermal forces and moments
Q_x , Q_y	Transverse shears resultants.
$Q^i{}_x$, $Q i_y$	Initial transverse shear resultants
t	Thickness of the plate
T, T0	Elevated and reference temperatures
u, v	Displacements of the mid-plane along x and y axes, respectively
$U_{i,}\ V_i, W_i$	Displacements of node i along x, y and z axes, respectively
w	Displacement along z axis
x, y, z	System of co-ordinate axes
Z_k, Z_{k-1}	Bottom and top distance of lamina from mid-plane

Greek

α	Shear correction factor
α_1, α_2	Thermal coefficients along 1 and 2 axes of a lamina, respectively
$\beta 1$, $\beta 2$	Moisture coefficients along 1 and 2 axes of a lamina, respectively
ε_x , ε_y, γ_{xy}	In-plane strains of the mid-plane.
$\varepsilon_x{}^N$, $\varepsilon_y{}^N$, $\gamma_{xy}{}^N$	Non-mechanical strains due to moisture and temperature
θ	Fiber orientation in a lamina
θ_x, θ_y	Rotations of the plate about x and y axes
ϑ_{12}, ϑ_{21}	Poisson's ratios
$\partial \partial x$, $\partial \partial y$	Partial derivatives with respect to x and y
ρ	Mass density
$(\rho)_k$	Mass density of kth layer from mid-plane
η, ξ	Local natural co-ordinates of an element
η_i, ξ_i	Local natural co-ordinates of the element at ith node
ϕ_x, ϕ_y	Shear rotations in x-z and y-z planes, respectively
ω_n	Natural frequency
Rx, Ry	The radii of curvatures in the x and y direction
Rxy	Radius of twist of the twisted plate
ϕ	Angle of twist of the twisted panel

Mathematical Operators

$[\ ]^{-1}$	Inverse of the matrix
$[\ ]^{T}$	Transpose of the matrix

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CHAPTER 1
INTRODUCTION

Composite materials are being increasingly used in aeronautical and aerospace industry, civil, naval and other engineering applications due to their light weight, high-specific strength and stiffness, excellent thermal characteristics easy in fabrication and other important specialties. Structures used in the above fields are more often exposed to high temperature as well as moisture. The varying environmental conditions due to moisture absorption and temperature seem to have an adverse effect on the stiffness and strength of the structural composites. This wide range of practical applications demands a fundamental understanding of their vibrations and static characteristics of laminated composite shell in different temperature and moisture concentration.

1.1 Importance of Present Study

In a weight sensitive application such as aerospace industries are intensively involved in the development of advanced turbomachinery composite materials because of their excellent properties. Composites are usually subjected to changing environmental conditions during their service life. Among different environmental conditions structures are more often exposed to high temperature and moisture. The effect of temperature is known as thermal effect and the effect of moisture absorption from the atmosphere is known as hygroscopic effect. The combined effects of temperature and moisture are known as hygrothermal effect. The varying environmental condition due to moisture absorption and temperature seem to have an adverse effect on stiffness and strength of the twisted composites. Heat gets conducted into the laminate when subjected to rise in the temperature. The laminate consumes moisture when subjected to the moist conditions. The swelling or expansion is more across the fibres of the lamina. Hygrothermal effects induce a dimensional change in the lamina. But the properties of the constituents of the laminate is contrast, its free movement is inhibited. Thus, deformations and corresponding stress conditions are induced. The induced hygrothermal stresses is referred as residual stresses. As, the matrix is more vulnerable to the hygrothermal condition than the fiber, the deformation is examined to be more in the transverse direction of the composite material. The rise in hygrothermal conditions decreases the elastic moduli of the material and induces internal initial stresses, which may affect the stability as well as the safety of the structures. Hence, it is necessary to study and analyze the behavior such as buckling and natural frequencies of laminated composite twisted panels due to the hygrothermal effect seem to be an important consideration in composite analysis and design, which are of practical interest.

Composite materials are being increasingly used in automotive, marine and especially weight sensitive aerospace applications, primarily because of the large values of specific strength and these can be tailored through the variation of fibre orientation and stacking sequence to obtain an efficient design. The optimum design of laminated structures demands an effective analytical procedure. But the presence of various coupling stiffness's and hygrothermal loading complicates the problem of vibration and buckling analysis of twisted panels for obtaining a suitable theoretical solution. Even the real situation of boundaries in laminated structures is more complex because there are many types of boundary conditions that can be called simply-supported or clamped edges. So a clear understanding about vibration and buckling characteristics of the composite twisted panels is of great importance. A comprehensive analysis of the vibration problems of homogeneous turbo machinery blades, modelled as beams has been studied exhaustively. Some studies available on the untwisted plates subjected to hygrothermal loading and some studies are there on vibration aspects of laminated composite pre twisted cantilever panels. The vibration and buckling analysis of laminated composite pre twisted cantilever panels subjected to hygrothermal loading is current case of study. The problem involves different complicated effect such as geometry, especially non-developable doubly curved surfaces, boundary conditions with variable temperature and moisture concentrations. The above discussed aspects need attention and thus constitute a problem of current interest.

A thorough review of earlier works done in this field is an important requirement to reach the objective and scope of the present investigation

CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

The vast uses of conventional metals, its alloys and the ever increasing demand of composite materials in plates and shells are the subject of research for many years. Though the investigations is mainly focused on vibration and buckling of composite twisted panels subjected to hygrothermal loading, some relevant researches on vibration & buckling of composite plate subjected to hygrothermal loading are also studied for the sake of its relevance and completeness. The literature reviewed in this chapter are grouped into two parts

- Twisted panels
- Composite plate

2.2 Reviews on twisted panels

The twisted panels have significant applications aerospace, civil, naval and other high-performance engineering applications due to their light weight, high specific strength, stiffness, excellent thermal characteristics, and ease in fabrication and other significant attributes. This range of practical applications demands a fundamental understanding of their vibration and buckling. Due to its significance, a large number of references deal with the free vibration of twisted plates. The following areas of analysis pertaining to the plates are covered in the review of literature:

2.2.1 Vibration and buckling of twisted panels subjected to hygrothermal load

The vibration analysis of turbomachinery blades has long been an intensive area of research. Its practical use can be seen in aerospace industry, fan and compressor blades. Knowledge of resonant frequencies and mode shape is necessary to ensure a risk free design of turbomachinery. To tailor the structural properties, fiber for reinforced composites laminates are increasingly used for designing turbomachinery blades requiring higher strength, more durability and less weight.

An excellent survey of the earlier works in the free vibration of turbomachinery blades was carried out by Rao [1973, 1977a, and 1980], Leissa [1980, 1981] for both stationary and rotating conditions. During the recent years, more rigorous methods of analysis have been developed based on the plate theory. Dokainish and Rawtani [2] investigated the natural frequencies and the mode shapes of a cantilevered plate mounted around a rotating disc. They considered the chord-wise bending effects, and obtain accurate results.

Ansari (1975) evaluated the nonlinear modes of vibration of a pretwisted non uniform cantilevered blade of unsymmetrical cross-section mounted on the periphery of a rotating disk. Kirkhope and Wilson (1976) calculated the coupled vibration modes of a rotating blade-disc system by using finite element method. Rao and Banerjee (1977) developed a polynomial frequency equation method to determine the natural frequencies of a cantilever blade with an asymmetric cross-section mounted on a rotating disc. Considering the blade as a discrete system, generalized polynomial expressions for the slope, linear and angular deflections are derived, using Myklestad expressions with necessary modifications.

Walker (1978) studied a conforming finite shell element suitable for the analysis of curved twisted fan blades and applied to a number of fan blade models. The element is assumed to be a doubly curved right helicoidally shell, in which the curvature is shallow with respect to the twisted base plane defining the helicoids.

Sreenwasamurthy and Ramamurti (1980) investigated the effect of a tip mass on the natural frequencies of a rotating Pre-twisted cantilever plate. Sreenwasamurthy and Ramamurti (1981) used a finite element technique to determine the natural frequency of a pre-twisted and tapered plate mounted on the periphery of a rotating disc. The pre-twisted plate has been idealized as an assemblage of three nodded rectangular shell elements with six degrees of freedom at each node. Leissa *et al*. (1983) determined the Vibrational characteristic of doubly curved shallow shells having rectangular plan forms, clamped along one edge and free on the other three using Ritz method with algebraic polynomial trial function.

In (1984) they experimented on vibration of twisted cantilever plates and summarized the previous and current studies. Ramamurti and Kielb (1984) presented a detailed comparison of the eigen frequencies of twisted rotating plates as obtained by using two different shape functions.

Rao and Gupta (1987) analyzed the free vibration characteristic of a rotating pre twisted small aspect ratio blade, mounted on a disc at a stagger angle, are determined using classical bending theory of thin shells.

Qatu and Leissa (1991) presented the vibration studies for laminated composite twisted cantilever plates using Ritz method with algebraic polynomial displacement function. Liew *et al*. (1994) presented a mathematical model to investigate the effects of initial twist on the vibratory characteristics of cantilever shallow conical shells. Pai and Nayfeh (1994) used a new approach to develop a geometrically exact non-linear beam model for naturally curved and twisted solid composite rotor blades undergoing large vibrations in three dimensional space. Lakhtakia (1995) studied wave propagation in a piezoelectric, continuously twisted, structurally chiral medium along the axis of spirality.

Liew *et al*. (1995) presented a computational investigation into the objects of initial twist and thickness variation on the vibratory characteristics of cantilevered pretwisted thin shallow conical shells with generally varying thickness. Rand (1995) presented a experimental method for detecting the natural frequencies of helicopter-like thin-walled rotating composite blades, and a study of their tendency in lamination angles and in the rotor angular velocity. Rand & Barkai (1997) studied a nonlinear formulation for the structural behavior of initially twisted solid and thin walled composite blades is presented. The model is designed to handle arbitrary thick solid cross-sections or general thin-walled geometries, and includes three-dimensional out-of-plane warping.

Parhi *et al*. (1999) investigated about the dynamic analysis of delaminated composite twisted plates based on a simple multiple delaminated models using finite element method. He *et al*. (2000) presented a paper about a computational method for characterizing the resonant frequency properties of cantilever pre-twisted plate composed of fibre-reinforced laminated composites.

Chen and Chen (2001) employed a finite element model to investigate the mean square response and reliability of a rotating composite blade with external and internal damping under stationary or non stationary random excitation. The effects of transverse shear deformation and rotary inertia are considered. Hu and Tsuiji (2001) investigated the vibration analysis of laminated cylindrical thin panels with twist and curvature using Rayleigh-Ritz method. Choi and Chou (2001) proposed a modified differential quadrature method (MDQM) for vibration analysis of elastically supported turbomachinery blades. Yoo *et al.* (2001) derived the equations of motion for the vibration analysis of rotating pre-twisted blades, derived from a modeling method which employs hybrid deformation variables.

Hu *et al.* (2002) studied a numerical method for free vibration of a rotating twisted and open conical shell by the energy method, based on a non-linear strain–displacement relationship of a non-rotating twisted and open conical shell on thin shell theory. Hu *et al.* (2002) investigated the vibration analysis of 7twisted conical shells with tapered thickness. Hu *et al*. (2002) studied a methodology for free vibration of a laminated composite conical shell with twist is proposed, in which a strain–displacement relationship of a twisted conical shell is given by considering the Green strain tensor on the general thin shell theory, the principle of virtual work is utilized, and the governing equation is formulated by the Rayleigh–Ritz procedure with algebraic polynomials in two elements as admissible displacement functions.

Zhu *et al.* (2002) examined the modeling of torsional vibration induced by extension-twisting coupling of anisotropic composite laminates with piezoelectric actuators. Lee *et al.* (2002) studied the finite element method based on the Hellinger Reissner principle with independent strain is applied to the vibration problem of cantilevered twisted plates and cylindrical, conical laminated shells. Kuang, Hsu (2002) studied the eigen value problem of a tapered pre-twisted orthotropic composite blade is formulated by employing the differential quadrature method (DQM). The Euler–Bernoulli beam model is used to characterize the pre-twisted orthotropic composite blade. Lim (2003) presented a new approach in the bending analysis of helicoidal structures with a large non-linear pre twist and an external lateral loading.

Yoo and Pierre (2003) investigated the modal characteristic of a rotating cantilever plate by using a dynamic modeling method for rectangular plates undergoing prescribed overall motion to derive the equations of motion. Oh *et al*. (2003) analysed the effects of pre twist and presetting on coupled bending vibrations of rotating thin-walled composite beams using refined dynamic theory of rotating blades modelled as anisotropic composite thin-walled beams, experiencing the flapping-lagging-transverse shear coupling. Lin *et al.* (2003) investigated about rotating no uniform pretwisted beams with an elastically restrained root and a tip mass Using Hamilton's principle derives the governing differential equations for the coupled bending–bending vibration of a rotating pretwisted beam with an elastically restrained root and a tip mass, subjected to the external transverse forces and rotating at a constant angular velocity. Chandiramani *et al.* (2003)

examined the free and forced vibration of a rotating, pretwisted blade modelled as a laminated composite, hollow (single celled), and uniform box-beam. The structural model includes transverse shear flexibility, restrained warping and centrifugal and Coriolis effects.

Nabi and Ganesan (2003) proposed the vibration characteristics of pre-twisted composite blades are analyzed using a three-noded triangular cylindrical shell element. The specific example of glass fibre reinforced plastic material is analyzed with its material damping. The effect of different parameters such as pre-twist, fibre orientation, skew angle, taper ratio and aspect ratio on natural frequency and system loss factor is investigated. Sakar and Sabuncu (2004) presented a finite element model for the static and dynamic stability study of a pretwisted aerofoil cross-section rotating blade subjected to an axial periodic force.

Tsai (2004) studied the Rotating vibration behavior of the turbine blades with different groups of blades. Hu *et al.* (2004) studied vibration of twisted plate Based on general shell theory and the first order shear deformation theory, an accurate relationship between strains and displacements of a twisted plate is derived by the Green strain tensor. Kee and Kim (2004) derived a general formulation for an initially twisted rotating shell structures including the effect of centrifugal force and Coriolis acceleration to study the vibration characteristic of initially twisted rotating shell type composite blades.

Hu *et al.* (2004) proposed vibration of an angle-ply laminated plate with twist considering transverse strain and rotary inertia, an analytical method by using Rayleigh-Ritz procedure. Dokainish and Rawtani (2005) used a finite element technique is to determine the natural frequencies and the mode shapes of a cantilever plate mounted on the periphery of a rotating disc. The plane of the plate is assumed to make any arbitrary angle with the plane of rotation of the disc. Chazly (2005) analyzed the Static and dynamic analysis of wind turbine blades using the finite element method.

Sahu *et al.* (2005) studied the vibration and stability behaviour of angle ply laminated twisted panels using Finite element method. Huang (2006) examined the effect of number of blades and distribution of cracks on vibration localization in a cracked pre-twisted blade system. Sahu *et al.* (2007) studied the buckling and vibration analysis of cross-ply laminated cantilever twisted plate using the finite element method with first order shear deformation theory. An eight noded isoparametric quadratic element is employed in the present analysis with five degrees of freedom per node.

Choi *et al*. (2007) studied the bending vibration control of the pre-twisted rotating composite thin-walled beam is studied. The formulation is based on single cell composite beam including a warping function, centrifugal force, Coriolis acceleration, pre-twist angle and piezoelectric effect. Hashemi *et al.* (2009) studied a finite element formulation for vibration analysis of rotating thick plates is developed. Mindlin plate theory combined with second order strain–displacement assumptions are applied for plate modelling.

Sinha and Turner (2011) proposed starting with the thin shell theory, the governing partial differential equation of motion for the transverse deflection of a rotating pre-twisted plate is derived to determine the natural frequencies of a twisted blade in a centrifugal field. Farhadi and Hasemi (2011) examined the aero elastic behavior of a supersonic rotating rectangular plate in the air medium using the Mindlin first-order shear deformation plate theory along with Von Korman nonlinear terms.

2.3 Reviews on Plates

The behaviour of structures subjected to in-plane loads with hygrothermal load is less understood in comparison with structures under transverse loads. Some of the literature covering composite plates subjected to hygrothermal loading is presented here.

2.3.1 Vibration of composite plates subjected to hygrothermal load

The effect of environment on the free vibration of laminated plates has been considered earlier by Whitney and Ashton (1971). They used the Ritz method to analyze symmetric laminates and equilibrium equations of motion in the case of antisymmetric angle-ply laminates, based upon the classical laminated plate theory. A few results were presented for only symmetric angle-ply laminates. Yang and Shieh [1987] considered vibrations of initially stressed antisymmetric cross-ply laminates. Initial stresses included both force and moment resultants. Rotary inertia and transverse shear effects were taken into account.

Dhanaraj and Palaninathan (1989) used the semi-loof shell element to study the free Vibrational characteristics of composite laminates under initial stress, which may also arise due to temperature. Results were presented showing how temperature affects the fundamental frequencies of anti-symmetric laminates. Sairam & Sinha (1992) are investigated the effects of moisture and temperature on the free vibration of laminated composite plates. The analysis is carried out by the finite element method with the quadratic isoparametric element, which takes transverse shear deformation into account. The analysis also accounts for lamina material properties at elevated moisture concentration and temperature. Results are presented showing the reduction in the natural frequency with the increase in uniform moisture concentration and temperature for symmetric and antisymmetric laminates with simply supported and clamped boundary conditions The vibration characteristics of rectangular plates subjected to non-uniform loading are studied using power series method by Khukla *et al.* (1995) & differential quadrature method by Gutierrez *et al.* (1999).

Lien- Wenchen, Chen (1988) studied vibrations of hygrothermal elastic composite plates. Parhi *et al.* (2001) presented a quadratic isoparametric finite element formulation based on the first order shear deformation theory for the free vibration and transient response analysis of multiple delaminated doubly curved composite shells subjected to a hygrothermal environment. Rohwer *et al*. (2001) investigated higher theories for thermal stresses in layered plates.

Cheng and Batra (2001) studied the effect of thermal loads on imperfectly bonded laminated composite shells, the interfacial imperfections necessitate that conditions requiring the continuity of surface tractions and displacements between adjoining faces be suitably modified, and interfacial damage properly accounted for. Shen and Shen (2001) examined the effect of hygrothermal conditions on the buckling and post buckling of shear deformable laminated cylindrical shells subjected to combined loading of axial compression and external pressure is investigated using a micro-to macro-mechanical analytical model.

Singha *et al.* (2001) analysed the thermal post buckling behaviour of graphite/epoxy multi-layered rectangular plates of various boundary conditions using the finite element method. Vel and Batra (2001) investigated the generalized plane strain quasi-static thermo elastic deformations of laminated anisotropic thick plates by using the Eshelby-Stroh formalism.

Shen and Shen (2001) studied the influence of hygrothermal effects on the post buckling of shear deformable laminated plates subjected to a uniaxial compression is investigated using a micro-to-macro-mechanical analytical model. Rutgerson and Bottega (2002) presented the buckling behavior of multilayer shells for composite structures subjected to combinations of uniform temperature change, applied external pressure, and applied and reactive circumferential edge loads. Wu and Chiu (2002) analyzed on thermally induced dynamic instability of laminated composite conical shells is investigated by means of a perturbation method. The laminated composite conical shells are subjected to static and periodic thermal loads.

Patel *et al.* (2002) studied the static and dynamic characteristics of thick composite laminates exposed to hygrothermal environment are studied using a realistic higher-order theory developed. Shen and Shen (2002) studied the effect of hygrothermal conditions on the buckling and post buckling of shear deformable laminated cylindrical panels subjected to axial compression is investigated using a micro-to macro-mechanical analytical model. Nonlinear vibration & dynamic response of simply supported shear deformable laminated plates on elastic foundations studied by Huang, Zheng (2003).

Nonlinear vibration & dynamic response of shear deformable laminated plates in hygrothermal condition studied by Zheng *et al.* (2004). Rao & Sinha (2004) studied dynamic response of multidirectional composites in hygrothermal environments. Jeyaraj *et al.* (2009) represented numerical studies on the vibration and acoustic response characteristics of a fiber-reinforced composite plate in a thermal environment by considering the inherent material damping property of the composite material. Initially the critical buckling temperature is obtained, followed by free and forced vibration analyses considering the pre-stress due to the imposed thermal environment. Panda & Singh (2009) presented nonlinear finite element model for geometrically large amplitude free vibration analysis of doubly curved composite spherical shell panel is presented using higher order shear deformation theory (HSDT).

2.3.2 Stability of composite plate subjected to hygrothermal load

The static stability of mechanical, civil engineering structures under compressive loading has always been a important field of research with the introduction of steel a century ago. The studies of temperature and moisture effects on the buckling loads of laminates are limited in number, and all these studies assumed perfectly initial configurations. Whitney and Ashton (1971) gave the first theoretical investigation of hygrothermal effects on the bending, buckling and vibration of composite laminated plates based on the classical laminated plate theory. The hygrothermal effects on the buckling of cylindrical composite panels were studied by Snead and Palazotto (1983) neglecting the transverse shear deformation effects, and by Lee and Yen (1989) including the transverse shear deformation effects.

Ram and Sinha (1992) studied the hygrothermal effects on the buckling of composite laminated plates using the finite element method. Chao and Shyu (1996) calculated the buckling loads for composite laminated plates under hygrothermal environments, where a micro to- macro-mechanical analytical model was proposed. These analyses addressed initial buckling problems and were based on the first-order shear deformation plate theory (FSDPT). Shen (2002) studied the effect of hygrothermal conditions on the buckling and post buckling of shear deformable laminated cylindrical panels subjected to axial compression which is investigated using a micro-to-macro-mechanical analytical model.

Eigen sensitivity analysis of moisture-related buckling of marine composite panels was studied by Barton (2007). He presented the elastic buckling of rectangular, symmetric angle-ply laminates subjected to a uniform moisture environment. This investigation presents an alternative method of computing the buckling load using Eigen values. Using this approach, an approximate closed-form expression is developed which can be used when exact solution are not available.
The studies on dynamic stability of structures are much less than in comparison to static stability & got a boost after Bolotin’s (1964) contribution to the literature. The instability results of thin simply supported (1995) plates are sparsely treated in the literature. The instability of laminated composite plates considering geometric non-linearity is also reported using finite element method by Balamurugan *et al.* (1996). The instability of the elastic plates considering geometric non-linearity is also reported, through a finite element formulation by Ganapathi (2000).

Partha & Singha (2006) studied the dynamic stability characteristics of simply supported laminated composite skew plates subjected to a periodic in-plane load are investigated using the finite element approach. The formulation includes the effects of transverse shear deformation, in-plane and rotary inertia. The boundaries of the instability regions are obtained using the Bolotin‟s method and are represented in the non-dimensional load amplitude-excitation frequency plane. Liu (2007) started a mesh-free formulation for the static and free vibration analyses of composite plates is presented via a linearly conforming radial point interpolation method. The radial and polynomial basis functions are employed to construct the shape functions bearing Delta function property.

2.4 Aim and scope of the present studies

A Review of literature shows that a lot of work has been done on the vibration and buckling of composite panels subjected to hygrothermal loading. Some work has been done on vibration of laminated composite twisted cantilever panels. However very few study is available on vibration and buckling of laminated composite twisted cantilever panels subjected to hygrothermal loading. The present study is mainly aimed at filling some of the lacunae that exist in the understanding of the vibration and stability characteristic of laminated composite twisted cantilever panels subjected to hygrothermal loading. The influence of various parameters such as angle of twist, number of layers, lamination sequence, ply orientation on the vibration and buckling behaviour of twisted panels is studied in detail.

Based on the review of literature, the different problems identified for the present investigation is presented as follows.

- Vibration of laminated composite twisted cantilever panels subjected to hygrothermal loading.
- Buckling of laminated composite twisted cantilever panels subjected to hygrothermal loading.

CHAPTER 3

THEORY AND FORMULATON

3.1 The Basic Problem

This chapter presents the mathematical formulation for vibration and buckling analysis of the twisted plate. The basic configuration of the problem considered here is a composite laminated doubly curved twisted panel of sides ‘a’ and ‘b’ as shown in Figure.

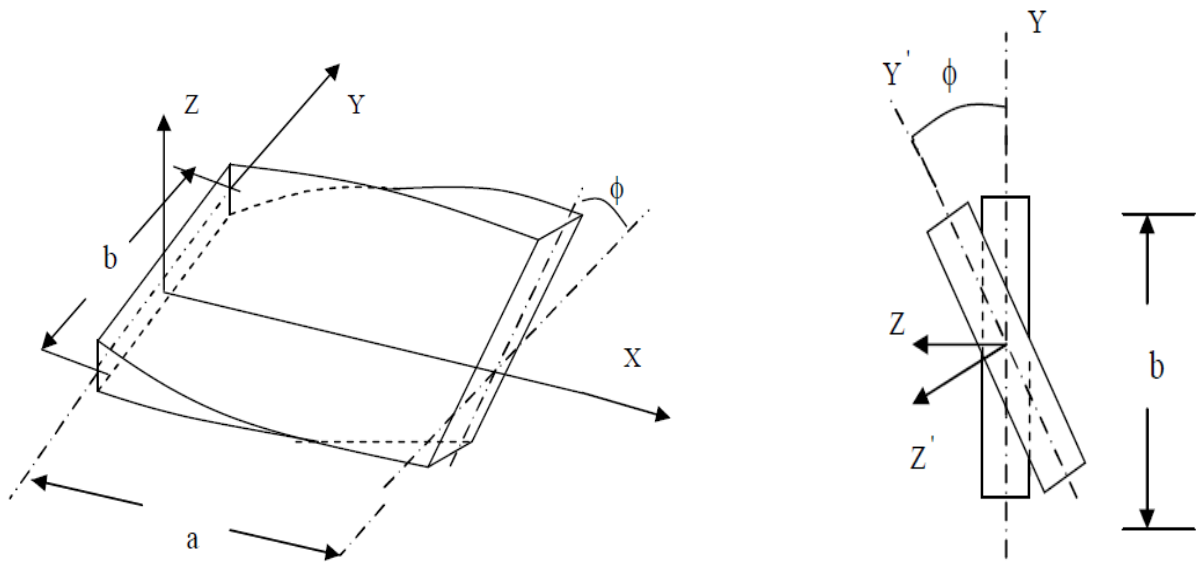
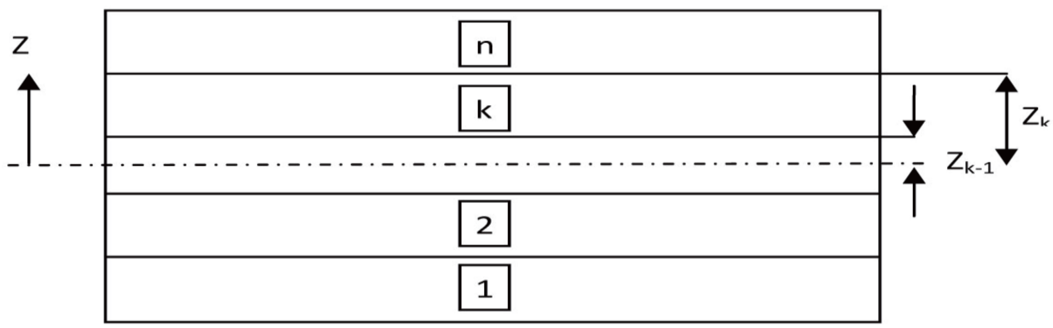


Fig. (a) Composite twisted panel

The twisted panel is modelled as a doubly curved panel with twisting curvature so that the analysis can be done for twisted plates, cylindrical and spherical configurations by changing the value of the curvature. The boundary conditions are taken to be that of a cantilever, that is fixed at the left end and free at the other edges. The basic composite twisted curved panel is considered to be composed of composite material laminates. ‘n’ denotes the number of layers of the laminated composite twisted panel.



(b) The lamination

Fig.1 Geometry of an N-layered laminate

3.2 Proposed Analysis

The governing equations for the vibration and buckling of laminated composite twisted Panels/shells subjected to in-plane loading are developed. The presence of external in-plane loads induces a stress field in the structure. This necessitates the determination of the stress field as a prerequisite to the solution of problems like vibration and buckling behaviour of pre twisted plates and shells. As the thickness of the structure is relatively smaller, the determination of the stress field reduces to the solution of a plane stress problem. The equation of motion represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The development of the regions of instability arises from Floquet's theory and the solution is obtained by Bolotin's approach using finite element method. The governing differential equations have been developed using the first order shear deformation theory (FSDT). The assumptions made in the analysis are given below.

3.3 Assumptions of the analysis

1) The analysis is linear with a few exceptions. This implies both linear constitutive relations (generalized Hooke's law for the material and linear kinematics) and small displacement to accommodate small deformation theory.

2) The composite twisted panels are of various shapes with no initial imperfections. The considerations of imperfections are less important for dynamic loading.

3) The straight line that is perpendicular to the neutral surface before deformation remains straight but not normal after deformation (FSDT). The thickness of the composite twisted panel

is small compared with the principal radii of curvature. Normal stress in the z-direction is neglected.

4) The loading considered is axial with a simple harmonic fluctuation with respect to time.

5) All damping effects are neglected.

3.4 Governing Equations

The governing differential equations, the strain energy due to loads, kinetic energy and formulation of the general dynamic problem are derived on the basis of the principle of potential energy and Lagrange's equation.

3.4.1 Governing Differential Equations

The equations of motion are obtained by taking a differential element of the twisted panel as shown in figure 2. This figure shows an element with internal forces like membrane forces N_x N_y and N_{xy}, shearing forces(Q_x and Q_y) and the moment resultants($M_x,\ M_y\ and\ M_{xy}$).

The governing differential equations for vibration of a shear deformable laminated composite plate in hygrothermal environment derived on the basis of first order shear deformation theory (FSDT) subjected to in-plane loads are (Chandrasekhar, Sahu and Dutta).

$$\frac{\partial Nx}{\partial x}+\frac{\partial Nxy}{\partial y}-\frac{1}{2}\left(\frac{1}{Ry}-\frac{1}{Rx}\right)\frac{\partial Mxy}{\partial y}+\frac{Qx}{Rx}+\frac{Qy}{Rxy}=P1\frac{\partial 2u}{\partial t}+\ P2\frac{\partial 2\theta x}{\partial t2}$$

$$\frac{\partial Nxy}{\partial x}+\frac{\partial Ny}{\partial y}+\frac{1}{2}\left(\frac{1}{Ry}-\frac{1}{Rx}\right)\frac{\partial Mxy}{\partial y}+\frac{Qy}{Rx}+\frac{Qx}{Rxy}=P1\frac{\partial 2v}{\partial t}+\ P2\frac{\partial 2\theta y}{\partial t2}$$

$$\frac{\partial Qx}{\partial x}+\frac{\partial Qy}{\partial y}-\frac{Nx}{Rx}-\frac{Ny}{Ry}-\ 2\frac{Nxy}{Rxy}+\ Nx^{\circ}\frac{\partial 2w}{\partial y2}+\ Ny^{\circ}\,\partial 2w/\partial y2=P1\frac{\partial 2w}{\partial t2}$$

$$\frac{\partial Mx}{\partial x}+\frac{\partial Mxy}{\partial y}-\ Qx=P3\frac{\partial 2\theta x}{\partial t2}+\ P2\frac{\partial 2u}{\partial t2}$$

$$\frac{\partial Mxy}{\partial x} + \frac{\partial My}{\partial y} - Qx = P3\frac{\partial 2\theta y}{\partial t2} + P2\frac{\partial 2v}{\partial t2} \quad (1)$$

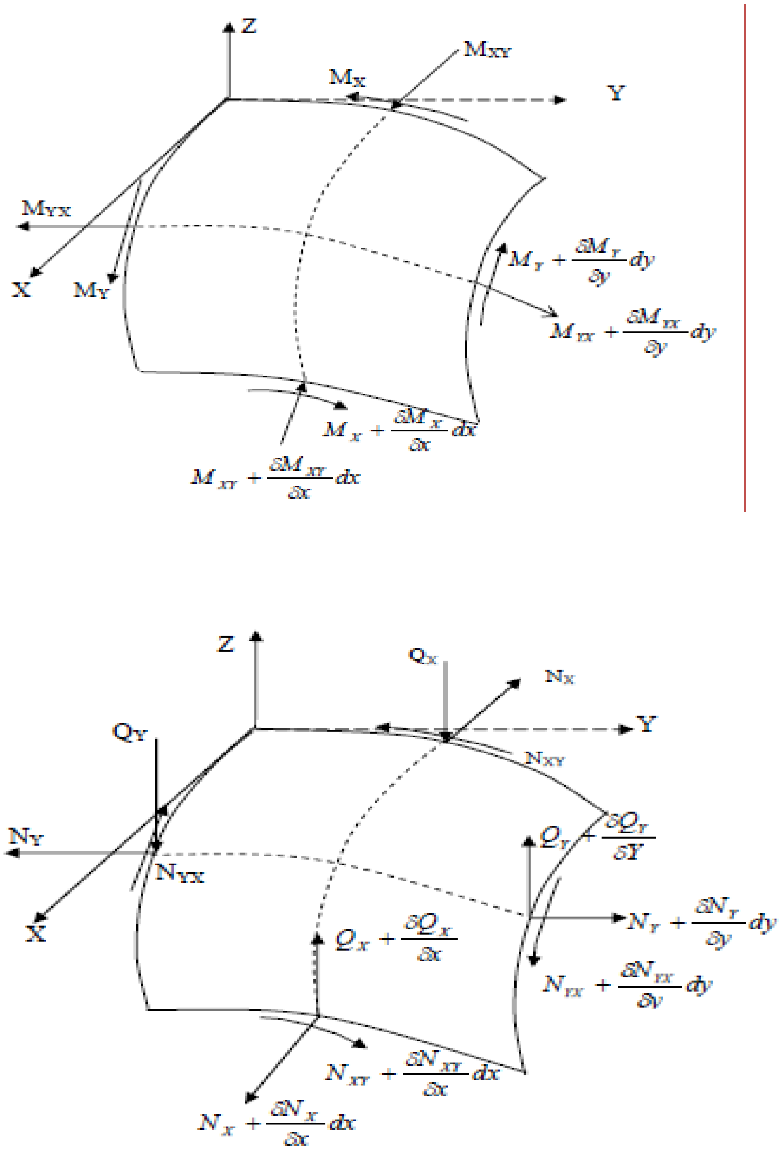


Figure 2: Force and moment resultants of the twisted panel

Where N_x^0 and N_x^0 are the external loading in the X and Y direction respectively Rx, Ry and Rxy identify the radii of curvatures in the x and y direction and radius of twist.

$$(P1, P2, P3) = \sum_{k=1}^{n} \int_{zk=1}^{zk} (\rho)k(1, z, z2)dz \quad (2)$$

Where n = number of layers of laminated composite curved panel, (ρ_k) = mass density of kth layer from mid-plane.

3.5 Finite Element Formulation

For problems involving complex geometrical and boundary conditions, analytical methods are not easily adaptable and numerical methods like finite element methods (FEM) are preferred. The finite element formulation is developed hereby for the structural analysis of isotropic as well as composite twisted panels using a curved shear deformable shell theory.

3.5.1 The shell element

The plate is made up of perfectly bonded layers. Each lamina is considered to be homogeneous and orthotropic and made of unidirectional fiber-reinforced material. The orthotropic axes of symmetry in each lamina are oriented at an arbitrary angle to the plate axes. An eight-noded isoparametric quadratic shell element is employed in the present analysis with five degrees of freedom u, v, w, θx and θy per node as shown in Figure. But the in-plane deformations u and v are considered for the initial plane stress analysis. The isoparametric element shall be oriented in the natural coordinate system and shall be transferred to the Cartesian coordinate system using the Jacobian matrix. In the analysis of thin shells, where the element is assumed to have mid-surface nodes, the shape function of the element is derived using the interpolation polynomial.

For problems involving complex in-plane loading and boundary conditions numerical methods like finite element method (FEM) are preferred. Eight-nodded isoperimetric element is used to the present free vibration problem. Five degrees of freedom u, v, w, θx and θy are considered at each node. The stiffness matrix, the geometric stiffness matrix due to residual stresses, geometric stiffness matrix due to applied in-plane loads and nodal load vector of the element are derived using the principle of minimum potential energy.

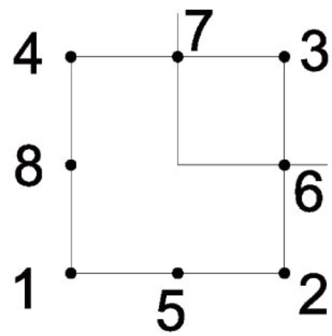


Fig. 3 Eight nodded isoparametric element

The element and displacement field are expressed by the shape function N_i

The shape function N_i $are\ defined\ as$

N_i= (1+ξξ_i)(1+ηη_i) (ξξ_i+ηη_i-1)/4 i=1 to 4 (3)

$N_i = (1-\xi^2)$ (1+ηη_i)/2 i= 5, 7 (4)

N_i= (1+ξξ_i) (1- η^2)/2 i = 6, 8 (5)

Where ξ and η are the local natural coordinates of the element and ξ_i and η_i are the values at ith node.

The derivatives of the shape function are N_i with respect to x and y are expressed in term of their derivatives with respect to ξ and η by the following relationship.

$$\begin{bmatrix} N_{i,x} \\ N_{i,y} \end{bmatrix} = [J]^{-1} \begin{bmatrix} N_{i,\xi} \\ N_{i,\eta} \end{bmatrix}$$

Where

$$[J]=\begin{bmatrix} X_{I,\xi} & Y_{I,\xi} \\ X_{i,\eta} & Y_{i,\eta} \end{bmatrix}$$

[J] is the Jacobian matrix. The shell with the initial stresses undergoes small lateral deformations. First order shear deformation theory is used and the displacement field assumes that the mid –

plane normal remains straight before and after deformation, but not necessarily normal after deformation, so that

u(x, y,z)= u_0(x,y)+ z θ_y(x,y)

v(x, y,z)= u_0(x,y)+ z θ_x(x,y)

w(x,y,z)= w_0(x,y)

Where u, v, w and u_0, v_0, w_0 are displacement in the x, y, z directions at any point and at the mid surface respectively. θx *and* θ_y are the rotations of the midsurface normal about the x and y axes respectively. Also

x =$\sum N_i x_i$, y =$\sum N_i y_i$

$u_0 = \sum N_i u_i$, $v_0 = \sum N_i v_i$, $w_0 = \sum N_i w_i$ (6)

$\theta_x = \sum N_i \theta_{xi}$, $\theta_y = \sum N_i \theta_{yi}$

3.5.2 Constitutive Relations

The constitutive relation for the twisted plate, when subjected to moisture and temperature,
are given by

{F} = [D] {ε} - {F^N} (7)

F^N = Non mechanical forces due to temperature and moisture
[*D*] and { ε } are Elasticity and strain matrix respectively.

Where
{F} = { N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y}^{T}
{F^N}= {Nx, Ny, Nxy, Mx, My, Mxy, Q_x, Q_y} ^{T}

$\{\varepsilon\}=\{\varepsilon x , \varepsilon y , \Upsilon xy , Kx , Ky , Kxy , \varphi x , \varphi y \}^T$

$$[D] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{112} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix}$$

Nx, Ny, Nxy = in-plane internal force resultants

Mx, My, Mxy = internal moment resultants.

Qx, Qy = transverse shear resultants.

Nx, Ny, Nxy = in-plane non mechanical force resultants due to moisture and temperature.

Mx, My, Mxy = non-mechanical moment resultants due to moisture and temperature.

ξ_x, ξ_y, ϒxy = in-plane strains of the mid-plane.

Kx, Ky, Kxy = curvature of the plate

ϕ_x, ϕ_y = shear rotations in X-Z and Y-Z planes respectively.

A_{ij}, B_{ij}, D_{ij} and Sij are the extensional, bending-stretching coupling, bending and transverse shear stiffnesses. They may be defined as

$$A_{ij} = \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^3 - z_{k-1}^3) \; ; i,j = 1,2,6$$

$$S_{ij} = \alpha \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1}) \; ; i,j = 4, \qquad (8)$$

And α is the transverse shear correction factor. The accurate prediction for anisotropic laminates depends on a number of laminate properties and is also problem dependent. A shear correction factor of 5/6 is used in the present formulation or all numerical computations.

z_k, z_{k-1} = bottom and top distance of lamina from mid-plane.

The non-mechanical force and moment resultants due to moisture and temperature are expressed as follows:

$$N_x^N, N_y^N, N_{xy}^N = \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})$$

$$M_x^N, M_y^N, M_{xy}^N = \frac{1}{2} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^2 - z_{k-1}^2) \text{ for } i, j = 1,2,6 \qquad (9)$$

Where

$$\{\xi\}_k = \{\xi_x^N, \xi_y^N, \xi_{xy}^N\} = [T]\{\beta_1, \beta_2\}_k^T (C - C_0) + T\{\alpha_1, \alpha_2\}_k^T (T - T_0)$$

in which $[T]$ = Transformation matrix due to moisture and temperature and is given as

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta \\ \sin^2\theta & \cos^2\theta \\ \sin 2\theta & -\sin 2\theta \end{bmatrix}$$

`

$\xi_x^N, \xi_y^N, \xi_{xy}^N$ =Non-mechanical strains due to moisture and temperature

β1, β2 = moisture coefficients along 1 and 2 axes of a lamina, respectively

α1, α2 = thermal coefficients along 1 and 2 axes of a lamina, respectively

T, T0 = elevated and reference temperatures

C, C0 = elevated and reference moisture concentrations

The laminated fiber reinforced shell is assumed to consist of a number of thin laminates as shown in figure. The principle material axes are indicated by 1 and 2 and modulli of elasticity of a lamina along these directions respectively. For the plane stress state, $\sigma_{0=0}$

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0\\ 0 & 0 & 0 & 0 & Q_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

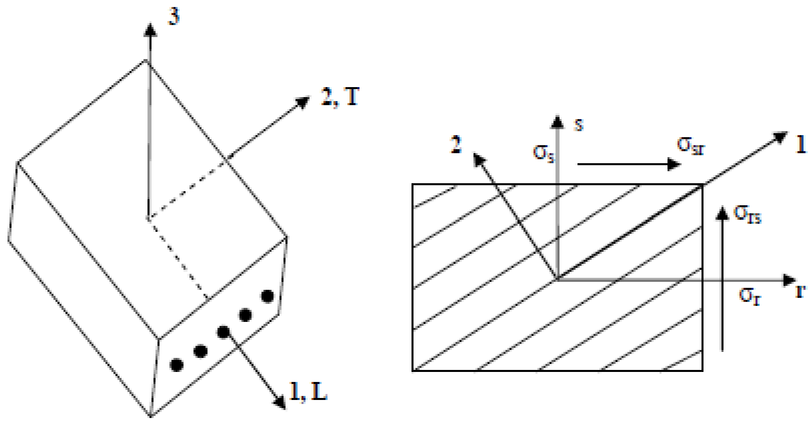


Figure 4: Laminated shell element

$(\bar{Q}_{ij})_k$ is defined as $(\bar{Q}_{ij})_k = [T_1]^T [Q_{ij}]_k [T_1]$

$$(\bar{Q}_{ij})_k = [T_1]^T [Q_{ij}]_k [T_1] \qquad \text{for i, j=1, 2, 6}$$

$$(\bar{Q}_{ij})_k = [T_2]\,(\bar{Q}_{ij})_k\,[T_2] \qquad \text{for i, j=4, 5} \tag{10}$$

Where

$$[T_1] = \begin{bmatrix}\cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta\\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}$$

$$[T_2] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$(\bar{Q}_{ij})_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \qquad \text{for i, j=1, 2 ,6}$$

$$(\bar{Q}_{ij})_k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \qquad \text{for i, j= 4, 5}$$

Where

$$Q_{11} = \frac{E_{11}}{(1 - v_{12} v_{21})}$$

$$Q_{12} = \frac{E_{11} v_{21}}{(1 - v_{12} v_{21})}$$

$$Q_{21} = \frac{E_{22}}{(1 - v_{12} v_{21})}$$

$$Q_{22} = \frac{E_{22}}{(1 - v_{12} v_{21})}$$

$$Q_{44} = G_{13}$$

$$Q_{55} = G_{23} \ , Q_{66} = G_{12}$$

E_1, E_2 = Young's moduli of a lamina along and across the fibers, respectively

$G_{12}, G_{13,} G_{23}$= Shear moduli of a lamina with respect to 1, 2 and 3 axes.

$\vartheta_{12}, \vartheta_{21}$ = Poisson's ratios

3.5.3 Strain Displacement Relations

Green-Lagrange's strain displacement relations are used throughout the structural analysis. The linear part of the strain is used to derive the elastic stiffness matrix and the nonlinear part of the strain is used to derive the geometric stiffness matrix. The total strain is given by

$$\{\xi\} = \{\xi_1\} + \{\xi_{nl}\} \tag{11}$$

The linear strain displacement relations for a twisted shell element are:

$$\xi_{xl} = \frac{\partial u}{\partial x} + \frac{w}{R_x} + zk_x$$

$$\xi_{yl} = \frac{\partial v}{\partial y} + \frac{w}{R_y} + zk_y$$

$$\gamma_{xyl} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} + zk_{xy}$$

$$\gamma_{xzl} = \frac{\partial w}{\partial x} + \theta_x - \frac{u}{R_x} - \frac{v}{R_{xy}}$$

$$\gamma_{yzl} = \frac{\partial w}{\partial y} + \theta_y - \frac{v}{R_y} - \frac{u}{R_{xy}} \tag{12}$$

Where the bending strains are expressed as

$$k_x = \frac{\partial \theta_x}{\partial x}, \; k_y = \frac{\partial \theta_y}{\partial y}$$

$$k_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{13}$$

The nonlinear strain components are defined as follows:

$$\varepsilon_{xnl} = \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right]$$

$$\varepsilon_{ynl} = \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right]$$

$$\gamma_{xynl} = \frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)\left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right) + z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right]$$

$$\gamma_{xznl} = (\frac{\partial u}{\partial x})\theta_y - (\frac{\partial v}{\partial x})\theta_x + z[\left(\frac{\partial \theta_y}{\partial x}\right)\theta_y + \left(\frac{\partial \theta_x}{\partial x}\right)\theta_x]$$

$$\gamma_{yznl} = (\frac{\partial u}{\partial y})\theta_y - (\frac{\partial v}{\partial y})\theta_x + z[\left(\frac{\partial \theta_y}{\partial y}\right)\theta_y + \left(\frac{\partial \theta_x}{\partial y}\right)\theta_x] \quad (14)$$

u, v = displacements of the mid-plane along x and y axes, respectively

w = displacement along z axis

θx, θy = rotations of the plate about x and y axes

3.5.4 Element Elastic Stiffness Matrix

The linear strain matrix {ε} is obtained by substituting equations (6) into (13), and is Expressed as

$$\{\varepsilon\} = [\mathrm{B}]\,\{d_e\} \quad (15)$$

Where

$$\{d_e\} = \{u_1 v_1 w_1 \theta_{x1} \theta_{y1} \ldots\ldots\ldots\ldots\ldots\ldots u_8 v_8 w_8 \theta_{x8} \theta_{y8}\}$$

$$[\mathrm{B}] = [[B_1]\,,[B_2]..\ \ldots\ldots\ldots\ldots\ldots\ldots[B_8]]$$

$$[B_i] = \begin{bmatrix} N_{i,}x & 0 & \frac{N_i}{R_x} & 0 & 0 \\ 0 & N_{i,}y & \frac{N_i}{R_y} & 0 & 0 \\ N_{i,}y & N_{i,}x & 2\frac{N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,}x & 0 \\ 0 & 0 & 0 & 0 & N_{i,}y \\ 0 & 0 & 0 & N_{i,}y & N_{i,}x \\ 0 & 0 & N_{i,}x & N_i & 0 \\ 0 & 0 & N_{i,}y & 0 & N_i \end{bmatrix} \quad \text{For i= 1, 2……8}$$

The elastic stiffness matrix is given by

$$[k_e] = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]|J|d\xi d\eta \tag{16}$$

3.5.5 Element Geometric Stiffness Matrix Due to Residual Stresses [K_{Ge}^{r}]

The non-linear strains, equations (10), are represented in matrix form as

$$\{\xi_{nl}\} = \{\xi_{xnl}, \xi_{ynl}, \gamma_{xynl}, \gamma_{yznl}\}^T = [R]\{d/\}/2 \tag{17}$$

Where

$$\{d\}=\{(\frac{\partial u}{\partial x}),(\frac{\partial u}{\partial y}),(\frac{\partial v}{\partial x}),(\frac{\partial v}{\partial y}),(\frac{\partial w}{\partial x}),(\frac{\partial w}{\partial y}),(\frac{\partial \theta x}{\partial y}),(\frac{\partial \theta y}{\partial x}),(\frac{\partial \theta x}{\partial x}),(\frac{\partial \theta y}{\partial y}),\theta x,\theta y\}^T$$

Equations $\{d\}$ may be expressed as:

$$\{d\}= [G]\{\{\delta e\} \tag{18}$$

Where

$$[G]=\begin{bmatrix} \frac{\partial N_i}{\partial_x} & & & & \\ \frac{\partial N_i}{\partial_y} & & & & \\ 0 & \frac{\partial N_i}{\partial_x} & & & \\ 0 & \frac{\partial N_i}{\partial_y} & & & \\ 0 & 0 & \frac{\partial N_i}{\partial_x} & & \\ 0 & 0 & \frac{\partial N_i}{\partial_y} & & \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial_x} & \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial_y} & \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial_x} \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial_y} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The geometric stiffness matrix due to residual stresses is given by**:**

$$[K_{Ge}{}^{r}] = \int_{-1}^{1}\int_{-1}^{1}[G]^{T}[S][G]|J|d\xi d\eta \tag{19}$$

where

$$[S]=\begin{bmatrix} S_{11} \\ S_{21} & S_{22} \\ 0 & 0 & S_{33} \\ 0 & 0 & S_{43} & S_{44} \\ 0 & 0 & 0 & 0 & S_{55} \\ 0 & 0 & 0 & 0 & S_{65} & S_{66} \\ 0 & 0 & S_{73} & S_{74} & 0 & 0 & S_{77} \\ 0 & 0 & S_{83} & S_{84} & 0 & 0 & S_{87} & S_{88} \\ S_{91} & S_{92} & 0 & 0 & 0 & 0 & 0 & 0 & S_{99} \\ S_{101} & S_{102} & 0 & 0 & 0 & 0 & 0 & 0 & S_{109} & S_{1010} \\ 0 & 0 & S_{113} & S_{114} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ S_{121} & S_{122} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In which

$S_{11} = S_{33} = S_{55} = N_x^i,$ $\quad S_{22} = S_{44} = S_{66} = N_y^i$ $\quad S_{21} = S_{43} = S_{65} = N_{xy}{}^{i}$

$S_{77} = S_{99} = N_x^i t^2/12$ $\quad S_{88} = S_{1010} = N_y^i t^2/12$ $\quad S_{877} = S_{109} = N_{xy}^i t^2/12$

$-S_{73} = S_{91} = M_x^i$ $\quad -S_{84} = S_{102} = M_y^i$ $\quad -S_{74} = -S_{83} = S_{92} = S_{101} = M^{i}{}_{xy}$

$S_{113} = S_{121} = Q_x^i$ $\quad -S_{114} = S_{122} = Q_y^i$

$N_x{}^i$, $N_y{}^i$, $N_{xy}{}^i$ =In-plane initial internal force resultants per unit length

$M_x{}^i$, $M_y{}^i$, $M_{xy}{}^i$ =initial internal moment resultants per unit length

$Q_x{}^i, Q_y{}^i$ = initial transverse shear resultant

3.5.6 Geometric Stiffness Matrix Due to Applied Loads [K_{Ge}^{a}]

The first three non-linear strain equations are represented in a matrix form:

$$\{\xi_{xnl}, \xi_{ynl}, \gamma_{xynl}\}^T = [U]\{f/\}/2$$

{ f } is expressed as:

$$\{f\} = U_x, U_y, V_x, V_y, W_x, W_y, \theta_{x,x}, \theta_{x,y}, \theta_{y,x}, \theta_{y,y}\}^T$$

$$\{f\} = [H]\{\delta_e\}$$

Where

$$[H] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 & \\ \frac{\partial N_i}{\partial y} & 0 & 0 & 0 & 0 & \\ 0 & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 & \\ 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & 0 & \\ 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & \end{bmatrix}$$

The geometric stiffness matrix due to applied in-plane loads is given by:

$$[K_{Ge}^{a}] = \int_{-1}^{1}\int_{-1}^{1}[H]^T[P][H]|J|d\xi d\eta \qquad (20)$$

Where

$$[P]=\begin{bmatrix} P_{11} & & & & & & & & & \\ P_{21} & P_{22} & & & & & & & & \\ 0 & 0 & P_{33} & & & & & & & \\ 0 & 0 & P_{43} & P_{44} & P_{55} & & & & & \\ 0 & 0 & 0 & 0 & P_{65} & P_{66} & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{77} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{87} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{88} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{99} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{109} & P_{1010} \end{bmatrix}$$

In which:

$P_{11} = P_{33} = P_{55} = N_x{}^a$

$P_{22} = P_{44} = P_{66} = N_y^a$

$P_{21} = P_{43} = P_{65} = N_{xy}^a$

$P_{77} = P_{99} = N_x{}^a t^2 / 12$

$P_{88} = P_{1010} = N_y^a t^2 / 12$

$P_{87} = P_{109} = N_{xy}^a t^2 / 12$

3.5.6 The Element Mass Matrix

$$[M_e] = \int_{-1}^{1}\int_{-1}^{1}[N]^T[P][N]|J|d\xi d\eta \tag{21}$$

Where the shape function matrix:

$$[N] = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \; i=1, 2.....8$$

$$[P] = \begin{bmatrix} P_1 & 0 & 0 & P_2 & 0 \\ 0 & P_1 & 0 & 0 & P_2 \\ 0 & 0 & P_1 & 0 & 0 \\ P_2 & 0 & 0 & P_3 & 0 \\ 0 & P_2 & 0 & 0 & P_3 \end{bmatrix}$$

In which the mass moment of inertia

$$P_1 = \sum_{1}^{n} \int_{e_{k-1}}^{e_k} \rho dz \text{ and } \mathrm{I}=\sum_{k=1}^{n} \int_{e_{k-1}}^{e_k} z^2 \rho dz$$

The element load vector due to external transverse static load q per unit area is given by

$$\{P_e\} = \iint N_i \begin{bmatrix} q \\ 0 \\ 0 \end{bmatrix} \mathrm{dxdy} \qquad (22)$$

Load Vector

The element load vector due to the hygrothermal forces and moments is given by:

$$\{ P_e^N \} = \int_{-1}^{1}\int_{-1}^{1}[N_j]^T \{q^a\}|j| d\zeta d\eta \qquad (23)$$

3.6 Solution Process

The element stiffness matrix, the initial stress stiffness matrix due to hygrothermal load, the mass matrix, geometric stiffness matrix due to applied loads and the load vectors of the element given by equations (16) and (19)-(23), are evaluated by first expressing the integrals in local Natural co-ordinates, ξ and η of the element and then performing numerical integration using Gaussian quadrature. Then the element matrices are assembled to obtain the respective global matrices [K], [M], $[K_{Ge}{}^{r}]$,$[K_{ge}{}^{a}]$,{P} and $\{P^N\}$.

The first part of the solution is to obtain the initial stress resultants induced by the external transverse static load and by moisture and temperature in static conditions

$$[K]\{\delta^i\} = \{P\} = \{P^N\} \qquad (24)$$

Then the initial stress resultants $N_x{}^i$, $N_y{}^i, N_{xy}{}^i$, $M_x{}^i$, $M_y{}^i, M_{xy}{}^i, Q_x{}^i, Q_y{}^i$ Qyi are obtained from equations (14) and (20). The second part of the solution involves determination of natural frequencies from the condition.

$$[[K] + [K_{Ge}{}^{r}]] - \omega_n^2 [M]\{q\} = 0$$

Where ω_n = Natural frequency

3.7 Computer Program

A computer program is developed by using MATLAB environment to perform all the necessary computations. The element stiffness and mass matrices are derived using a standard procedure. Numerical integration technique by Gaussian quadrature is adopted for the element matrix. Since the stress field is non-uniform, plane stress analysis is carried out using the finite element techniques to determine the stresses and these stresses are used to formulate the geometric stiffness matrix. The overall matrices [K], [Kg], and [M] are obtained by assembling the corresponding element matrices, using skyline technique. The boundary conditions are imposed restraining the generalized displacements in different nodes of the discretized structure.

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Introduction

The composites plates/shells with arbitrary geometries and boundary conditions subjected to hygrothermal loading got important roles to play as the structural elements in aerospace and other engineering structures. The plate and shell structures subjected to hygrothermal loading cause non-uniform stress field which greatly affects the stability and dynamic behavior of structures.

The present chapter deals with the results of the analysis of the vibration and buckling characteristics of homogeneous and laminated composite twisted cantilever panels subjected to hygrothermal loading using the formulation given in the previous chapter. As explained, the eight-node isoparametric quadratic shell element is used to develop the finite element procedure. The first order shear deformation theory is used to model the twisted panels considering the effects of transverse shear deformation. The vibration and buckling studies are carried out for laminated composite twisted panels subjected to hygrothermal loads to consider the effect of various parameters. The studies in this chapter are presented as follows:

- Convergence study
- Comparison with previous studies
- Numerical results

4.2 Boundary conditions

The clamped (C) boundary condition of the laminated composite twisted panel using the first order shear deformation theory is:

u = v = w = θ_x = θ_y = 0 at the left edge.

4.3 Vibration and buckling of twisted panels

- Convergence study
- Comparison with previous results
- Numerical results

4.3.1 Convergence study

The convergence study is done for non-dimensional frequencies of free vibration of cantilever square 4 layers symmetric cross ply and symmetric angle ply laminated composite plates for elevated temperature and moisture conditions for different mesh division as shown in Table 1 and table 2 for three angle of twist(ϕ =0^0, 15^0, 30^0). The study is further extended to buckling analysis of laminated composite twisted cantilever plates subjected to hygrothermal condition as presented in Table 3 and 4 and this mesh is employed throughout free vibration, buckling and dynamic stability analysis of laminated composite plates in hygrothermal environment.

Table 1: Convergence of non-dimensional free vibration frequencies for cantilever twisted plate for different ply orientations at 325K temperature

a/b=1, a/t=25, At T=300K, E_1=130x10^9, E_2 =9.5x10^9, G_{12}=6x10^9, $G_{13}=G_{12}$, $G_{23}=0.5G_{12}$

υ_{12} = 0.3, α_1=-0.3x10^{-6}/^{0} K, α_2 = 28.1x10^{-6}/^{0}K

Non-dimensional frequency, $\varpi = \omega_n a^2 \sqrt{\dfrac{\rho}{E_2 t^2}}$

Mesh	Non-dimensional frequencies of free vibration for different angles of twist and ply orientation at 325K Temperature								
	0/90/90/0			45/-45/-45/45			30/-30/-30/30		
	0^0	15^0	30^0	0^0	15^0	30^0	0^0	15^0	30^0
4*4	0.8485	0.8303	0.7690	0.1596	0.1838	0.1166	0.5700	0.6130	0.5923
8*8	0.8484	0.8302	0.7689	0.1527	0.1782	0.1084	0.5664	0.6104	0.5901
10*10	0.8483	0.8302	0.7689	0.1517	0.1774	0.1072	0.5658	0.6100	0.5897

Table 2: Convergence of non-dimensional free vibration frequencies for cantilever twisted plate for different ply orientations at 0.1% moisture concentration

a/b=1, a/t=25, At C=0.00, E_1=130x10^9, E_2=9.5x10^9, G12=6x10^9, G13=G12, G23=0.5G12

υ_{12}=0.3, α_1= -0.3x10^{-6}/^{0} K, α_2 = 28.1x10^{-6}/^{0}K

$$\text{Non-dimensional frequency, } \varpi = \omega_n a^2 \sqrt{\frac{\rho}{E_2 t^2}}$$

Mesh	Non-dimensional frequencies of free vibration for different angles of twist and ply orientation at 325K Temperature								
	0/90/90/0			45/-45/-45/45			30/-30/-30/30		
	0^0	15^0	30^0	0^0	15^0	30^0	0^0	15^0	30^0
4*4	0.8779	0.8612	0.8053	0.3045	0.3158	0.2869	0.6022	0.6345	0.6135
8*8	0.8777	0.8611	0.8052	0.3007	0.3124	0.2835	0.5990	0.6320	0.6113
10*10	0.8777	0.8611	0.8052	0.3001	0.3118	0.2830	0.5984	0.6316	0.6110

Table 3: Convergence of non-dimensional critical loads for cantilever twisted plate for different ply orientations at 325K temperature.

a/b=1, a/t=25, At C=0.00, E_1=130x10^9, E_2=9.5x10^9, G12=6x10^9, G13= G12, G23=0.5 G12

υ_{12} =0.3, α_1= -0.3x10^{-6}/^{0} K, α_2 = 28.1x10^{-6}/^{0}K

$$\text{Non-dimensional buckling load } \lambda = (N_x b^2)/E_2 h^3$$

Mesh	Non-dimensional frequencies of free vibration for different angles of twist and ply orientation at 325K Temperature								
	0/90/90/0			45/-45/-45/45			30/-30/-30/30		
	0^0	15^0	30^0	0^0	15^0	30^0	0^0	15^0	30^0
4*4	1.9561	1.8305	1.4589	0.0574	0.0755	0.0290	.7879	0.9158	0.8196
8*8	1.9558	1.8303	1.4587	0.0527	0.0712	0.0252	0.7803	0.9099	0.8148
10*10	1.9558	1.8303	1.4586	0.0520	0.0706	0.0246	0.7790	0.9090	0.8141

Table 4: Convergence of non-dimensional critical loads for cantilever twisted plate for different ply orientations at 0.1% moisture concentration

a/b=1, a/t=25, At C=0.00, E_1=130x10^9, E_2=9.5x10^9, G12=6x10^9, G13= G12, G23=0.5 G12

υ_{12} =0.3, α_1= -0.3x10^{-6}/^{0} K, α_2 = 28.1x10^{-6}/^{0}K

Mesh	Non-dimensional frequencies of free vibration for different angles of twist and ply orientation at 325K Temperature								
	0/90/90/0			45/-45/-45/45			30/-30/-30/30		
	0^0	15^0	30^0	0^0	15^0	30^0	0^0	15^0	30^0
4*4	2.1049	1.9793	1.6077	0.2117	0.2292	0.1804	0.8871	0.9886	0.8843
8*8	2.1046	1.9791	1.6075	0.2105	0.2250	0.1765	0.8800	0.9828	0.8794
10*10	2.1046	1.9791	1.6075	0.2098	0.2243	0.1760	0.8789	0.9820	0.8788

4.3.2 Comparison with previous studies

After the convergence study, the accuracy and efficiency of the present formulation are established through comparison with previous studies. The results obtained by this formulation are compared with the analytical, independent finite element and/or experimental results published by other investigators wherever possible for variety of problems on twisted plates and shells. Unless otherwise mentioned the mesh division used in this present analysis is 8x8 considering the whole plate/shell for almost all the geometrical configurations, based on convergence studies.

4.3.2.1 Vibration of composite plates and shells subjected to hygrothermal environment

The present finite element formulation is validated for free vibration analysis of laminated composite twisted panels as shown in table 5. The non dimensional frequency calculated by the present formulation is compared with non-dimensional free vibration frequencies for cantilever twisted plates for different ply orientations presented by Qatu and Leissa (1991) and He *et al.* (2000) in Table 5. This shows good agreement between the present study and those of Qatu and Leissa (1991) and He *et al.* (2000).

Table 5: Comparison of non-dimensional free vibration frequencies for cantilever twisted plates for different ply orientations

a/b=1, E_1=138x10^9, E_2=8.96x10^9, G_{12}=7.1x10^9, $G_{13}=G_{12}=G_{23}$

υ_{12}=0.3, α_1=-0.3x10^{-6}/^{0} K, α_2 = 28.1x10^{-6}/^{0}K, Angle of each plies= (θ,-θ, θ)

(a) Angle of twist (ϕ) =15^0

$$\text{Non-dimensional frequency, } \varpi = \omega_n a^2 \sqrt{\frac{\rho}{E_2 t^2}}$$

Ply Orientation θ	Non-dimensional free vibration frequencies for cantilever twisted plates for different ply orientations					
	b/h=100			b/h=20		
	He *et al.* (2000)	Qatu and Leissa (1991)	Present study	He *et al.* (2000)	Qatu and Leissa (1991)	Present study
0	1.0034	1.0035	1,0029	1.0031	1.0031	0.9883
15	0.92938	0.9296	0.9285	0.89791	0.8981	0.8824
30	0.74573	0.7465	0.7449	0.68926	0.6899	0.6764
45	0.52724	0.5286	0.5270	0.47810	0.4790	0.4694
60	0.35344	0.3545	0.3539	0.33374	0.3343	0.3296
75	0.27208	0.2723	0.2724	0.26934	0.2695	0.2681
90	0.25544	0.2555	0.2554	0.25540	0.2554	0.2574

(b) Angle of twist (ϕ) =45^0

$$\text{Non-dimensional frequency, } \varpi = \omega_n a^2 \sqrt{\frac{\rho}{E_2 t^2}}$$

Ply Orient-ation θ	Non-dimensional free vibration frequencies for cantilever twisted plates for different ply orientations					
	b/h=100			b/h=20		
	He *et al*. (2000)	Qatu and Leissa (1991)	Present study	He *et al*. (2000)	Qatu and Leissa (1991)	Present study
0	0.86072	0.8605	0.8601	0.85848	0.8585	0.8434
15	0.80404	0.8045	0.8025	0.79397	0.7941	0.7768
30	0.65337	0.6549	0.6513	0.63529	0.6358	0.6194
45	0.46703	0.4698	0.4665	0.44783	0.4488	0.4364
60	0.31218	0.3125	0.3116	0.30115	0.3019	0.2959
75	0.23488	0.2352	0.2348	0.23300	0.2332	0.2316
90	0.21919	0.2192	0.2192	0.21900	0.2190	0.2183

4.3.2.2 Buckling of composite plates and shells subjected to hygrothermal environment

The present finite element formulation is validated for buckling analysis of laminated composite twisted panels as shown in table 6. The non dimensional buckling load calculated by the present formulation is compared with non-dimensional buckling load for cantilever twisted plates for different ply orientations and different angle of twist presented by Sahu *et al.* (2005) in Table 6. The present finite element results show good agreement with the previous numerical results published in the literature for buckling of composite plates subjected to hygrothermal load.

Table 6: Comparison of non-dimensional buckling load for cantilever twisted plates for different ply orientations and angle of twist

a/b=1, E_1=138x10^9, E_2 =8.96x10^9, G_{12}=7.1x10^9, $G_{13} = G_{12} = G_{23}$

υ_{12}=0.3, α_1=-0.3x10^{-6}/^{0} K, α_2 = 28.1x10^{-6}/^{0}K,

Angle of twist (φ)	Ply Orientation(θ)	Sahu et. al (2005)		Present Study	
		b/h=20	b/h=100	b/h=20	b/h=100
	0	2.998	3.179	3.1312	3.1787
	15	2.226	2.162	2.111	2.1611
	30	1.275	1.139	1.1091	1.1393
0^0	45	0.647	0.573	0.5621	0.5752
	60	0.322	0.315	0.3113	0.3158
	75	0.235	0.224	0.2234	0.2246
	90	0.199	0.260	0.2054	0.206
30^0	0	2.504	2.544	2.4971	2.544
	15	1.889	2.195	2.0413	2.1952
	30	1.233	1.412	1.2233	1.411
	45	0.584	0.703	0.5794	0.7032
	60	0.278	0.315	0.2771	0.3156
	75	0.182	0.187	0.1822	0.1871
	90	0.164	0.164	0.1643	0.165

4.3.3 Numerical Results

After obtaining the convergence study and validating the formulation with the existing literature, the results for vibration and buckling studies for the twisted plate subjected to temperature and moisture are presented. The material properties used for the numerical study:

At T= 300 K, E_1=138x10^9, E_2=8.96x10^9, G_{12}=7.1x10^9, $G_{13}=G_{12}$, G23=0.5 G12

υ_{12}=0.3, α_1=-0.3x10^{-6}/^{0} K, α_2 = 28.1x10^{-6}/^{0}K, β_1=0, β_2=0.44

4.3.3.1 Vibration results for plate

4.3.3.1.1 For angle ply

The variation of non-dimensional frequency parameter Non-dimensional frequency,

$\varpi = \omega_n a^2 \sqrt{\dfrac{\rho}{E_2 t^2}}$ for a eight layer laminated composite plate subjected to uniform distribution of temperature from 225K to 375K is shown in fig.5. With increase in temperature the frequency decreases for different twisted cantilever plates. As the temperature increases from 225K to 375K the decrease in non dimensional frequency for ply orientations 0^0, 15^0 and 30^0 is about 16.18%, 25.3% and 69.86 %. respectively.

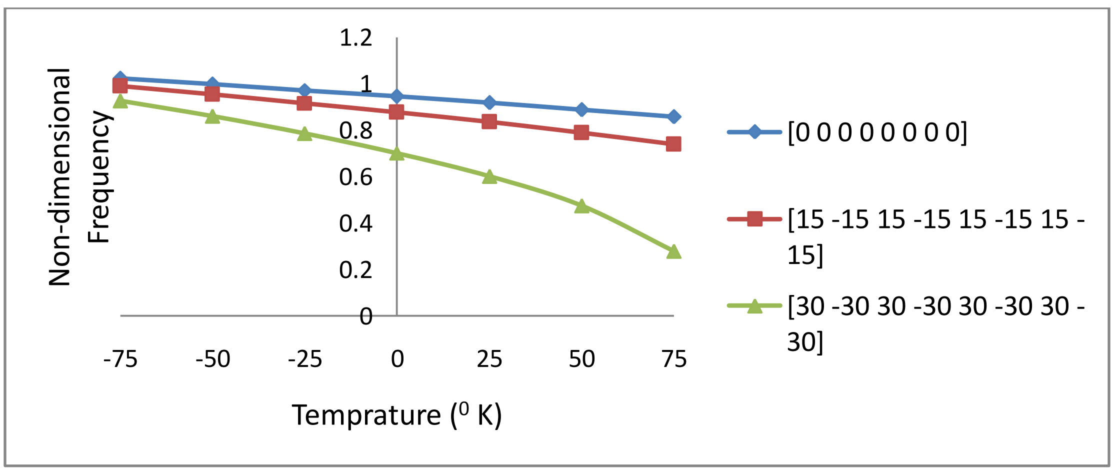


Fig 5: Effect of temperature on non-dimensional frequency for ϕ =30^0 and for angle-ply laminated pre twisted cantilever plates (a /b=1, b/t=20)

The variation of non-dimensional frequency of vibration of pre twisted cantilever laminated anti symmetric angle ply plates with moisture is presented in figure 6. As the moisture concentration increases from 0% to 0.4%, the decrease in non-dimensional frequency is about 12.23% for 0^0, about 16.02% for 15^0 ply orientations and for 30^0 ply orientations the % decrease in the non dimensional frequency due to increase in moisture concentration is about 47.58%.

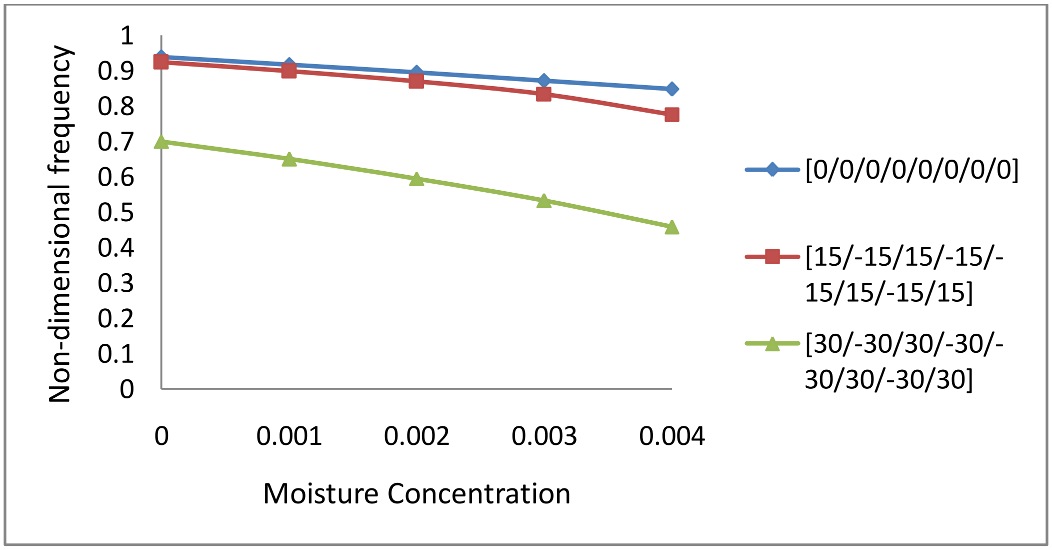


Fig 6: Effect of moisture on non-dimensional frequency for $\phi = 30^0$ and for angle-ply laminated pre twisted cantilever plates (a /b=1, b/t=20)

4.3.3.1.2 For Cross-ply

The variation of non-dimensional frequency of vibration of pre twisted cantilever laminated symmetric and anti symmetric cross ply plates with increasing temperature from 225K to 375K is presented in figure 7. The percentage decrease is about 56.45% for anti-symmetric cantilever cross-ply laminates and the decrease is about 69.40% for symmetric cantilever cross-ply laminates. Result shows that In case of cross-ply plates decrease in frequency for symmetric cross-ply plates is more than anti symmetric cross-ply plates.

The variation of non-dimensional frequency of vibration of pre twisted cantilever laminated symmetric and anti symmetric cross ply plates with moisture is presented in figure 8.

The percentage decrease is about 53.7% for anti-symmetric cantilever cross-ply laminates and 40.74% for symmetric cross-ply laminates.

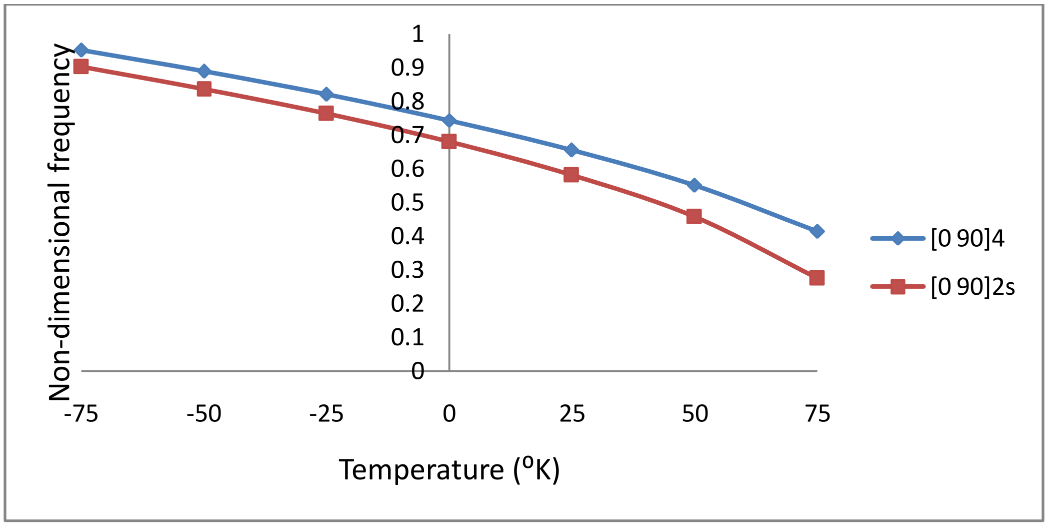


Fig.7: Effect of temperature on non-dimensional frequency for ϕ =30^{0}and for cross-ply laminated pre twisted cantilever plates (a /b=1, b/t=20)

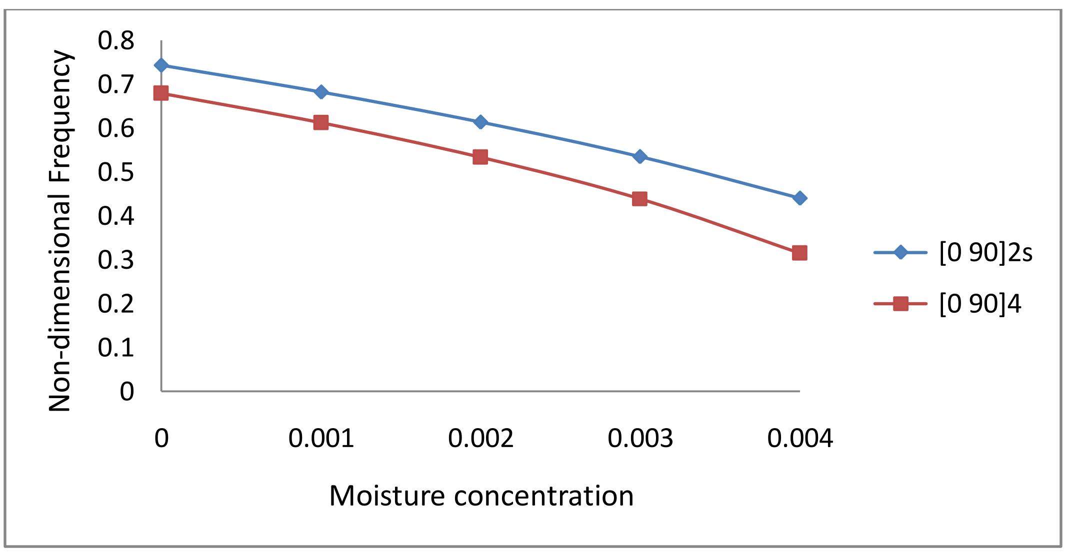


Fig 8: Effect of moisture on non-dimensional frequency for Φ =30^0 and for cross ply laminated pre twisted cantilever plates (a /b=1, b/t=20)

4.3.3.2 Effect of angle of twists on vibration of pre twisted cantilever plates

The variation of non dimensional frequency with increase in temperature for symmetric angle ply laminated twisted cantilever plate (30/-30/30/-30)s for different angles of twist (ϕ) is shown in figure 9.When the temperature increases to 375K the decrease in frequency for angle of twist 0^0 is about 74.83%, for twist angles 15^0 and 30^0 the percentage decrease in non dimensional frequency with increase in temperature is about 11.25% and 10.6% respectively.

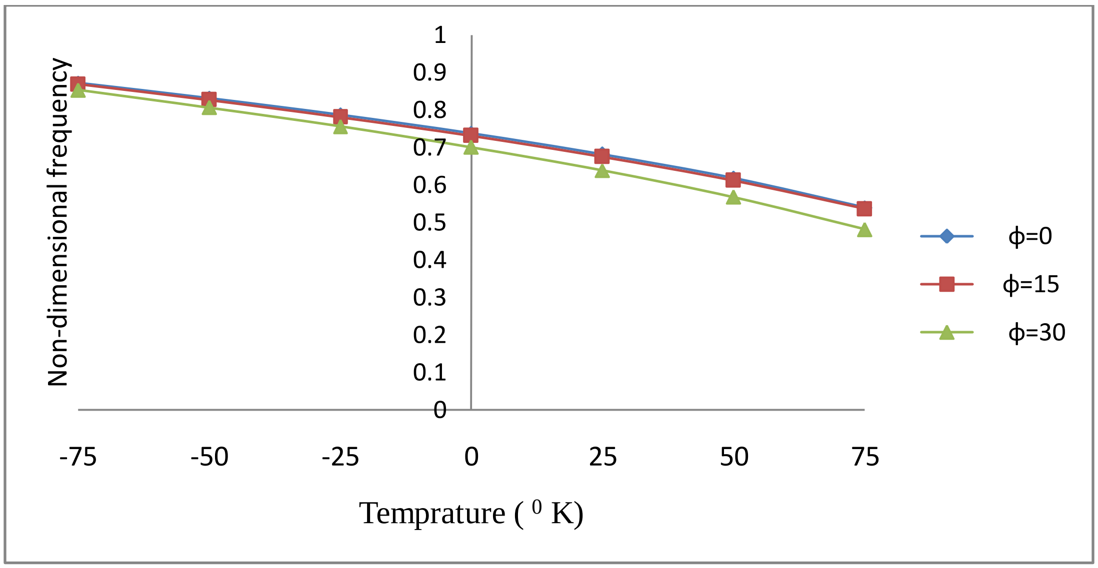


Fig 9: Effect of temperature on non-dimensional frequency for laminated angle ply pre twisted plate (30/-30/30/-30)s for different angles of twist (a/b=1, b/t=20)

The variation of non dimensional frequency with increase in moisture for symmetric laminated twisted cantilever plate (30/-30/30/-30) for different angles of twist (ϕ) is shown in figure 10.when the moisture increases to 0.4% the decrease in frequency for angle of twist 0^0,15^0 and 30^0 is about 71.42%. , 16.29% and 16.67% respectively.

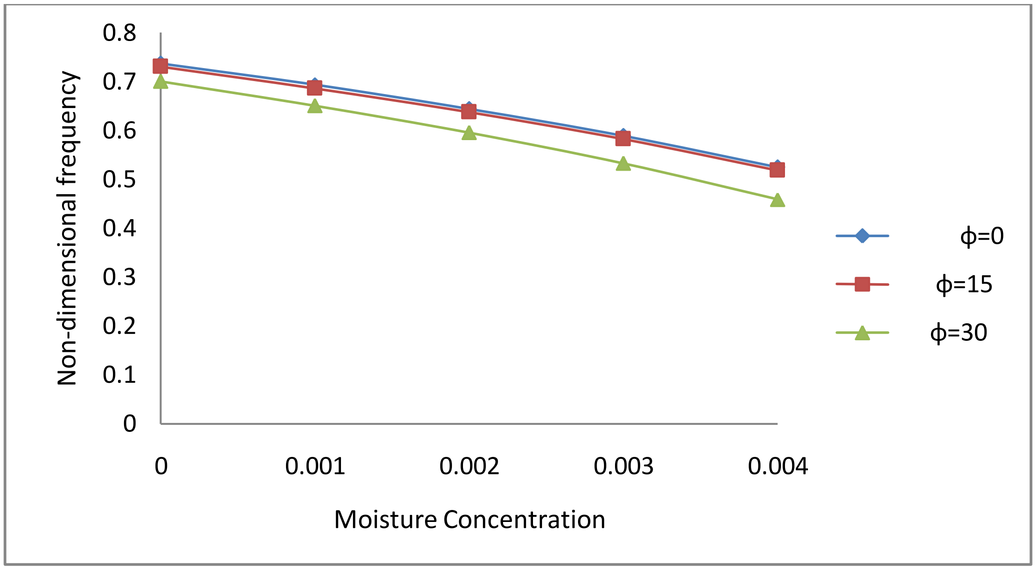


Fig 10:Effect of moisture on non-dimensional frequency for laminated angle ply pre twisted plate (30/-30/30/-30) for different angle of twist (a/b=1, b/t=20)

The variation of non dimensional frequency with increase in temperature for symmetric cross ply laminated twisted cantilever plate (0/90/90/0) for different angles of twist (ϕ) is shown in figure 11.When the temperature increases to 375K the decrease in frequency for angle of twist 0^0 ,15^0 and 30^0 are 55.91%,47.02% and 56.45% respectively. It shows that decrease in frequency for angle of twist 0^0 is more than the decrease for twist angle 15^0.

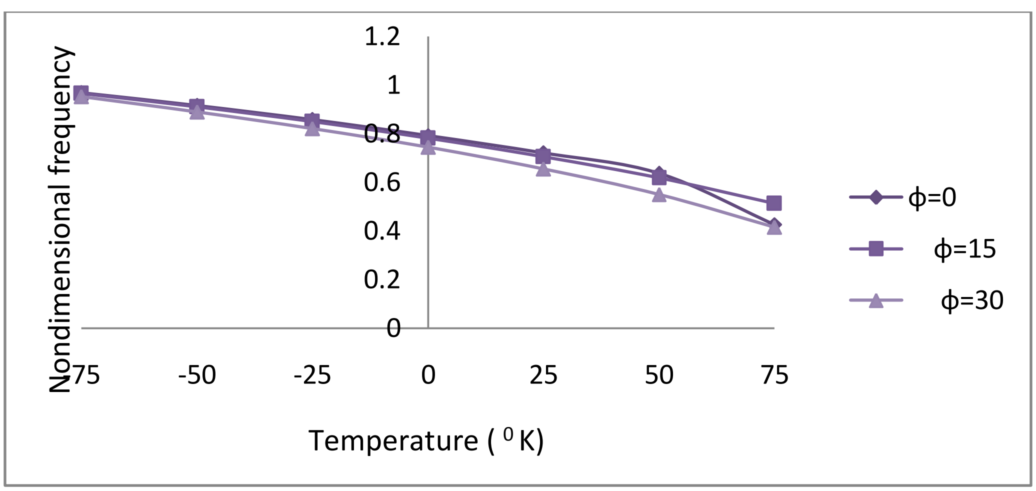


Fig 11:Effect of temperature on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/90/0) for different angle of twist (a/b=1, b/t=20)

In fig (12) When moisture increases to 0.4% the frequency decreases for angle of twist o^0 is 2.91% and for angle 30^0 decreases to 3.10%.

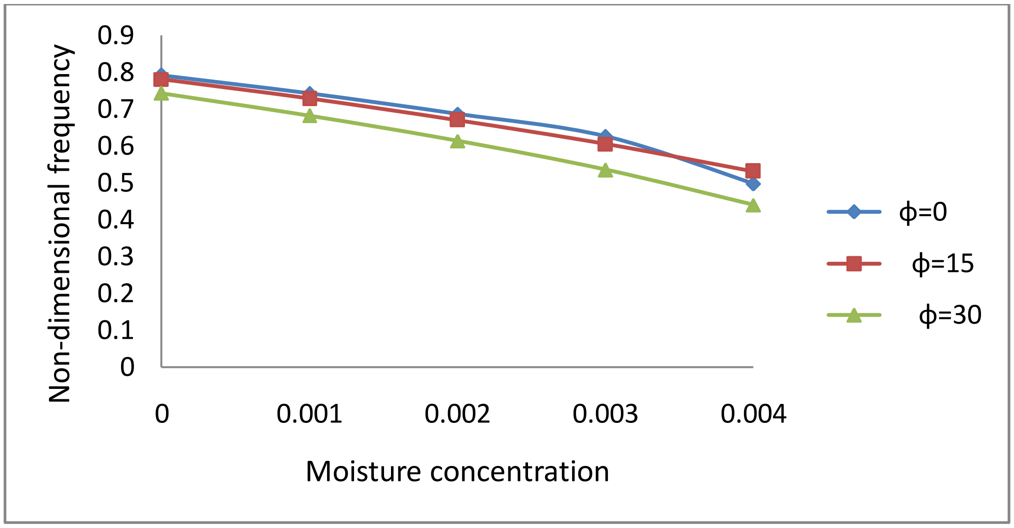


Fig.12:Effect of moisture on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/90/0) for different angles of twist (a/b=1, b/t=20)

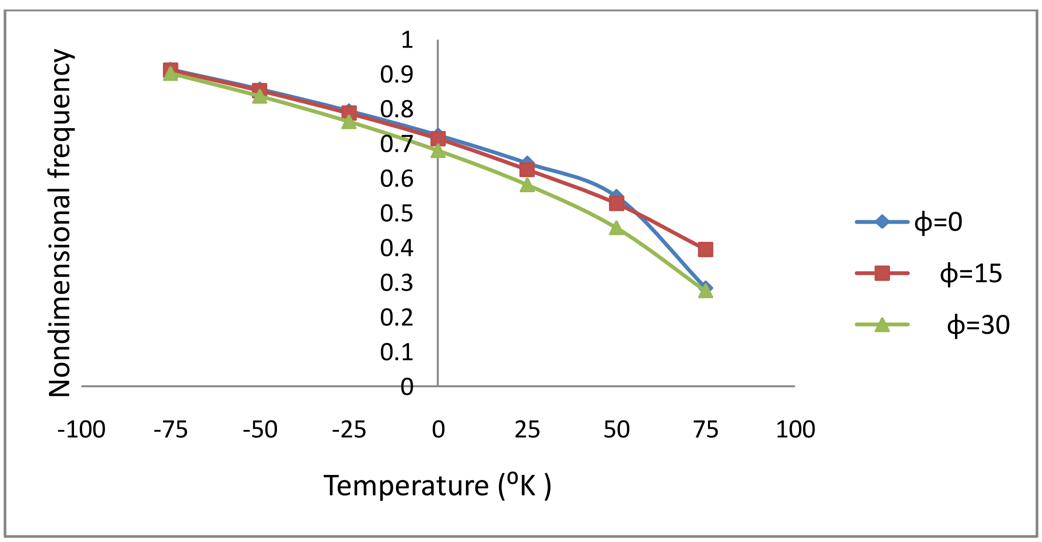


Fig 13: Effect of temperature on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90) for different angle of twist (a/b=1, b/t=20)

The effect of temperature on non-dimensional frequency for laminated anti symmetric cross ply pre twisted cantilever plate (0/90/0/90) for different angle of twist (ϕ) is shown in figure 13. As the temperature increases from 225K to 375K, the decrease in non dimensional frequency for angle of twist 0^0,15^0 and 30^0 is about 69%., 56.16% and 69.4% respectively. This result shows that decrease in frequency for angle of twist 0^0 and 30^0 are nearly same.

The effect of moisture on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90) for different angles of twist (ϕ) is shown in figure 14. The decrease in non dimensional frequency for angle of twist 0^0 and 15^0 and 30^0 is about 3.47%, 3.71% and 4.71% respectively. This result shows that decrease in frequency for angle of twist 0^0, $15^0$and 30^0 are nearly same

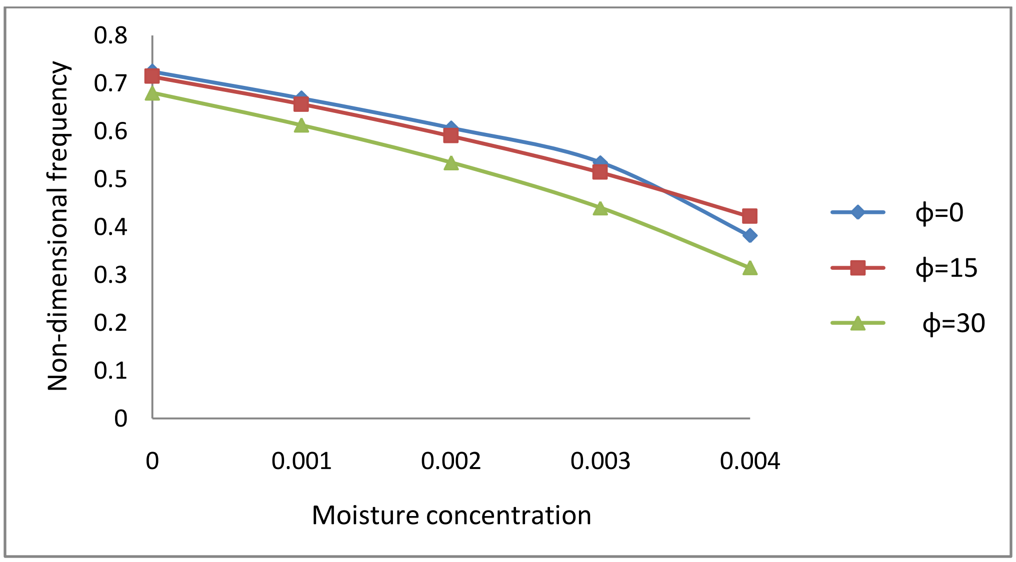


Fig 14:Effect of moisture on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90) for different angles of twist (a/b=1, b/t=20)

4.3.3.3 Effect of change in aspect ratio on vibration of plate

- **a/b=3**

The variation of non-dimensional frequency for a eight layer laminated composite plate subjected to uniform distribution of temperature is shown in fig.15. As the temperature increases from 225K to 375K the decrease in non dimensional frequency for ply orientations 0^0, 15^0 and 30^0 is about 4.37 %, 4.50 % and 5.38 %. respectively. Result shows that with increase in temperature the frequency decrease for different ply orientation.

The variation of non-dimensional frequency of vibration of pre twisted cantilever laminated symmetric angle ply plates with moisture are shown in figure 16. When moisture concentration increases from 0% to 0.4% the decrease in non-dimensional frequency is about 1.865 % for 0^0 ply orientation and about 2.55 % for 15^0 ply orientation and 2.42% for 30^0 ply orientations.

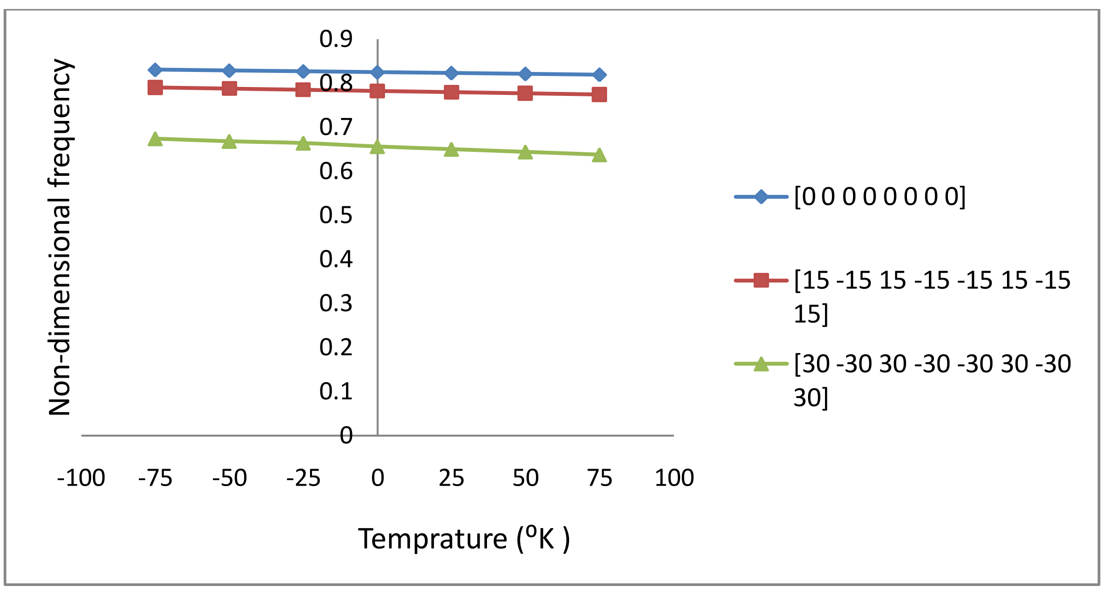


Fig 15:Effect of temperature on non-dimensional frequency for ϕ =30^0 and for angle-ply laminated pre twisted cantilever plates (a /b=3, b/t=20)

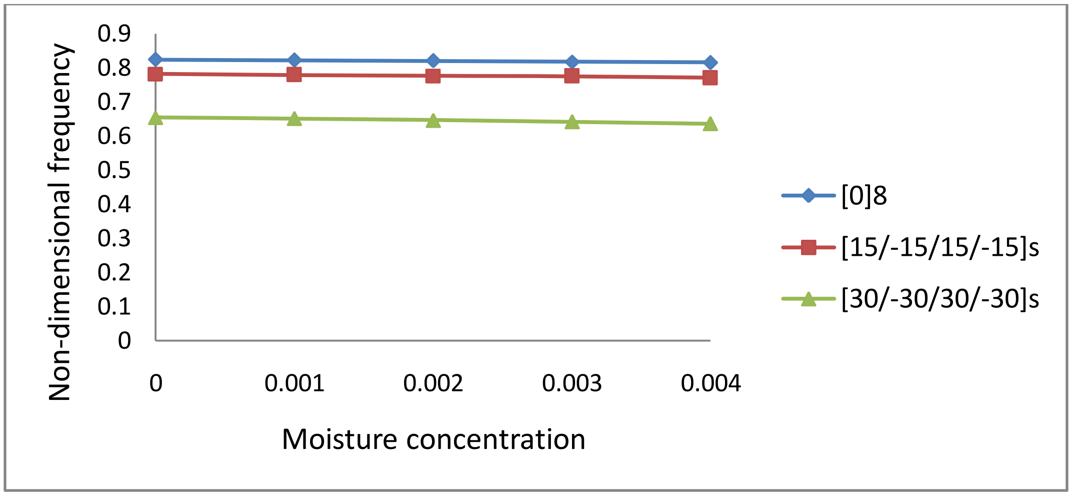


Fig 16:Effect of moisture on non-dimensional frequency for ϕ =30^0 and for angle-ply laminated pre twisted cantilever plates (a /b=3, b/t=20)

The variation of non-dimensional frequency of vibration of pre twisted cantilever laminated symmetric and anti symmetric cross ply plates with temperature is presented in figure 17. The variation of non-dimensional frequency of vibration with increase in temperature for for symmetric cross -ply cantilever plate is 6.25% and for anti-symmetric cross ply plate, it is decreased up to 7.37%. In case of cross-ply plates decrease in frequency for symmetric cross-ply plates is less than anti-symmetric cross-ply plates.

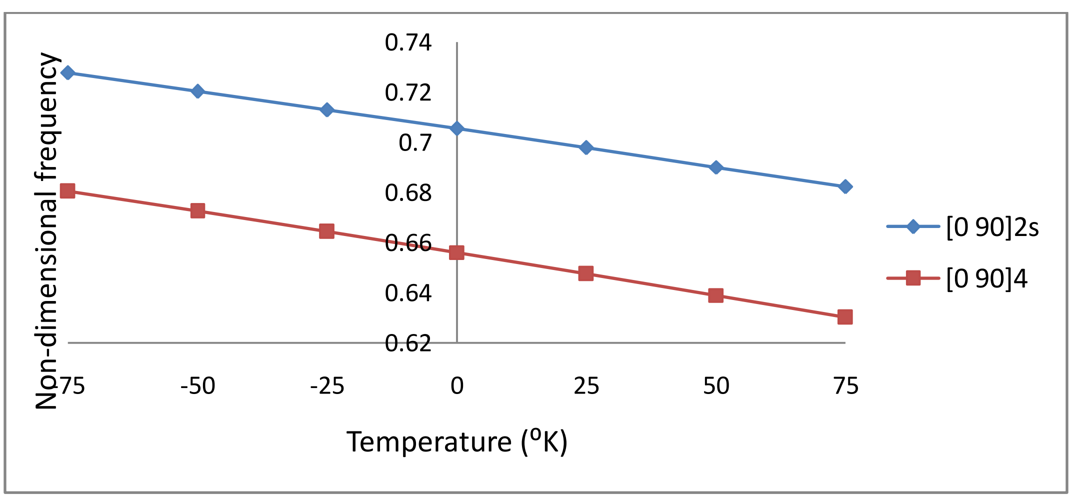


Fig 17:Effect of temperature on non-dimensional frequency for ϕ =15^0 and for cross-ply laminated pre twisted cantilever plates (a /b=1, b/t=20)

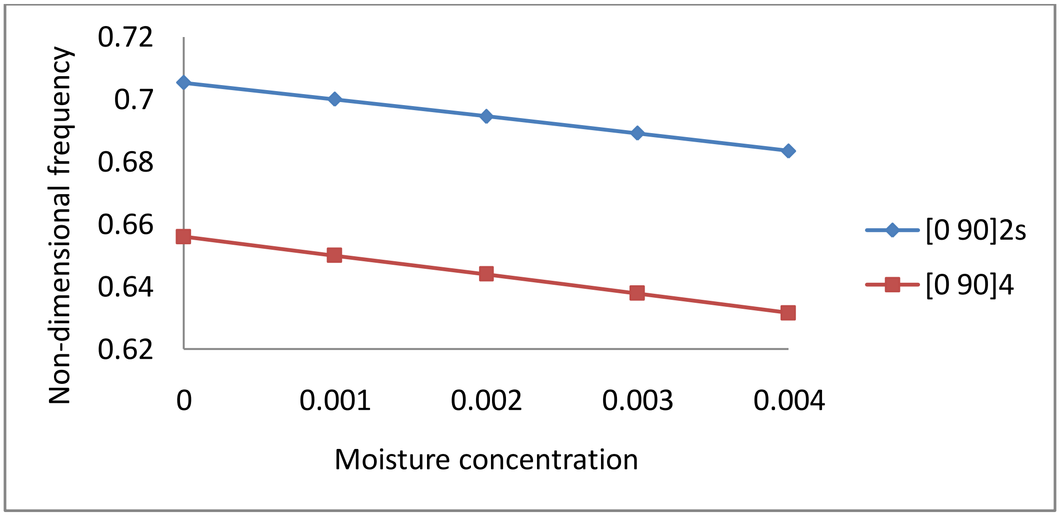


Fig 18:Effect of moisture on non-dimensional frequency for Φ =15^0 and for cross ply laminated pre twisted cantilever plates (a /b=3, b/t=20)

4.3.3.4 Effect of angle of twists on vibration of pre twisted cantilever plates

The variation of non dimensional frequency with increase in temperature for symmetric angle ply laminated twisted cantilever plate (30/-30/30/-30)s for different angles of twist (ϕ) is shown in figure 19. With increase in temperature from 225K to 375K the frequency decreases for different twisted cantilever plates are 4.37 %, 4.50% respectively. 5.38%

The variation of non-dimensional frequency of vibration of pre twisted cantilever laminated antisymmetric angle ply plates with moisture is presented in figure 20. As the moisture concentration increases from 0% to 0.4%, the decrease in non-dimensional frequency isabout 9.93% for 0^0, about 16.28% for 15^0 ply orientations and for 30^0 ply orientations the % decrease in the non dimensional frequency due to increase in moisture concentration is about 41.88%.

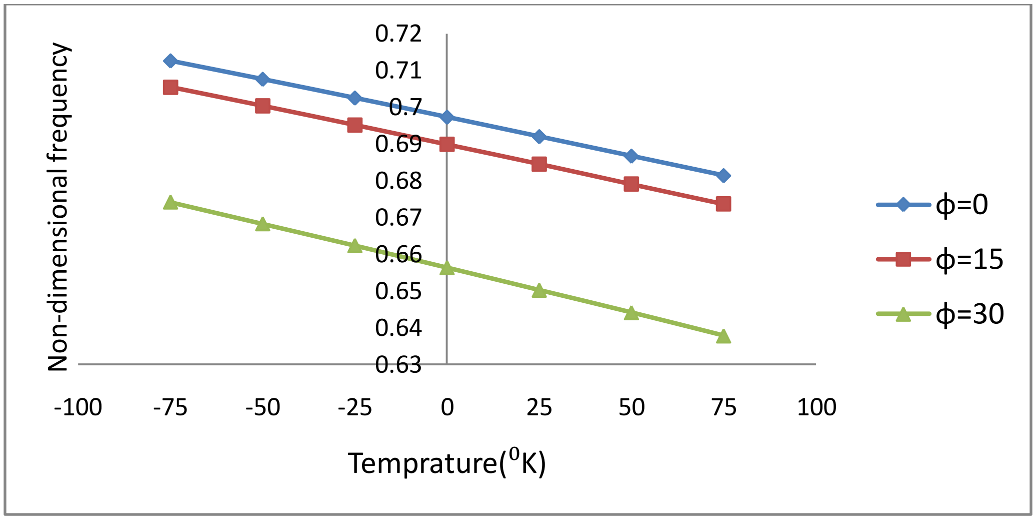


Fig19: Effect of temperature on non-dimensional frequency for laminated angle ply pre twisted plate (30/-30/30/-30)s for different angles of twist (a/b=3, b/t=20)

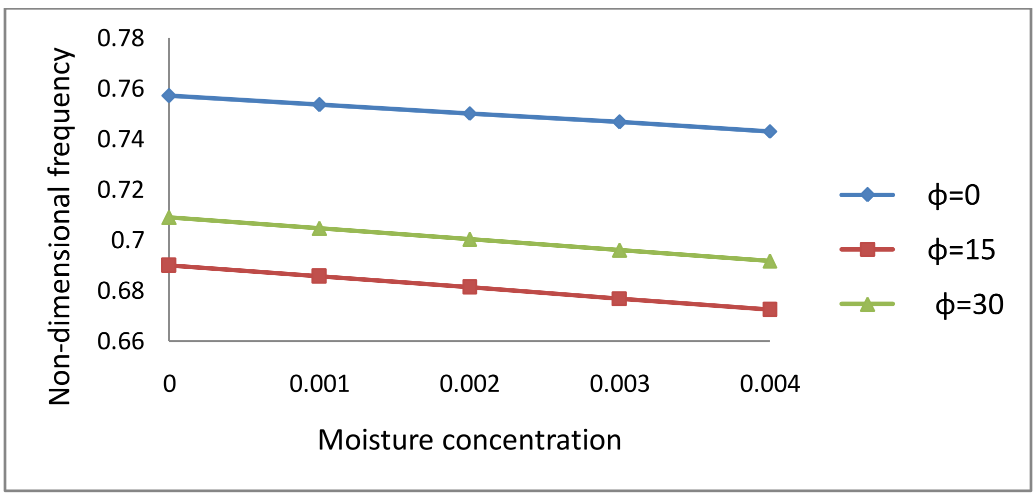


Fig 20:Effect of moisture on non-dimensional frequency for laminated angle ply pre twisted cantilever plate (30/-30/30/-30)s for different angle of twist (a/b=3, b/t=20)

The effect of temperature on non-dimensional frequency for laminated anti symmetric cross ply pre twisted cantilever plate (0/90/0/90)s for different angle of twist (ϕ) is shown in figure 21. The percentage decrease for angle of twist 0^0,15^0 and 30^0 is about 6.93%., 7.37% and 8.98% respectively. This result shows that decrease in frequency for angle of twist 0^0 and 30^0 are nearly same.

The effect of moisture on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90)s for different angles of twist (ϕ) is shown in figure 22. As the moisture increases the percentage decrease in non dimensional frequency for angle of twist 0^0, 15^0 and 30^0 is about 3.74% , 3.71% and 4.56% respectively. This result shows that decrease in frequency for angle of twist 0^0, $15^0$and 30^0 are almost same.

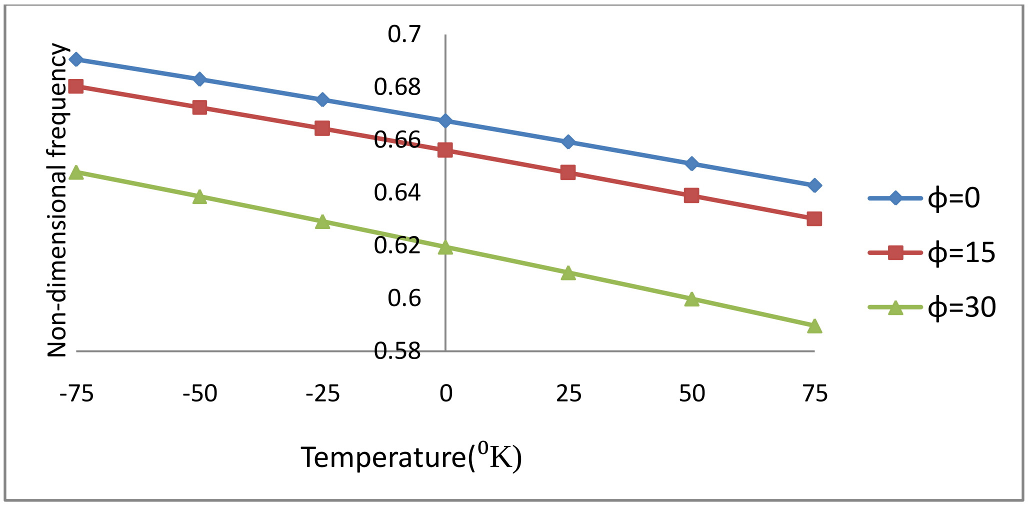


Fig 21 Effect of temperature on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90)s for different angle of twist (a/b=3, b/t=20)

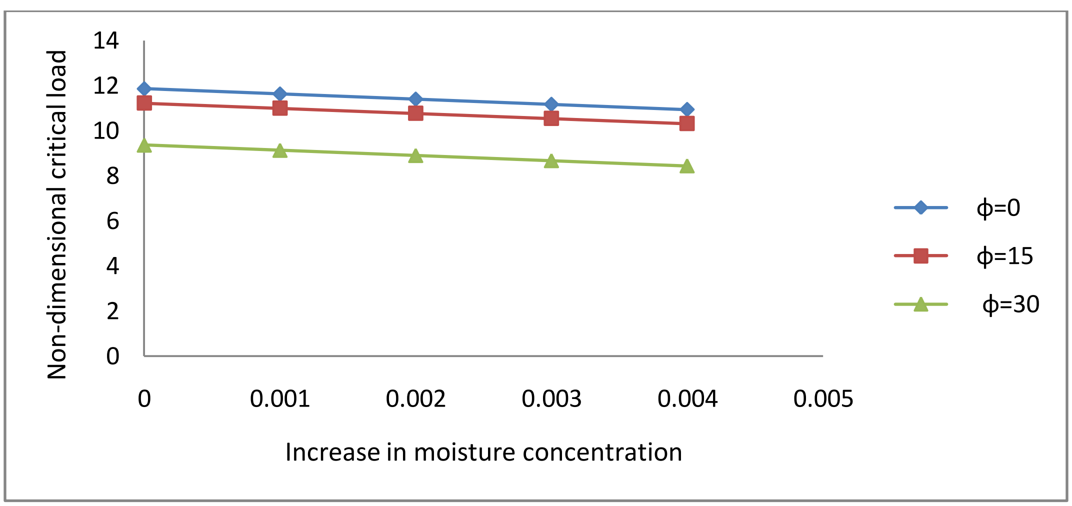


Fig 22 Effect of moisture on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90)s for different angle of twist (a/b=3, b/t=20)

The variation of non dimensional frequency with increase in temperature for symmetric cross ply laminated twisted cantilever plate (0/90/90/0) for different angles of twist (ϕ) is shown in figure 23.When the temperature increases to 375K the decrease in frequency for angle of twist 0^0 and 15^0 are 5.87%, 6.25 % and respectively. It shows that the percentage decrease for both angle of twist is nearly same.

In fig (24) when moisture increases to 0.4% the frequency decreases for angle of twist o^0 is 2.91% and for angle 30^0 decreases to 3.10%.

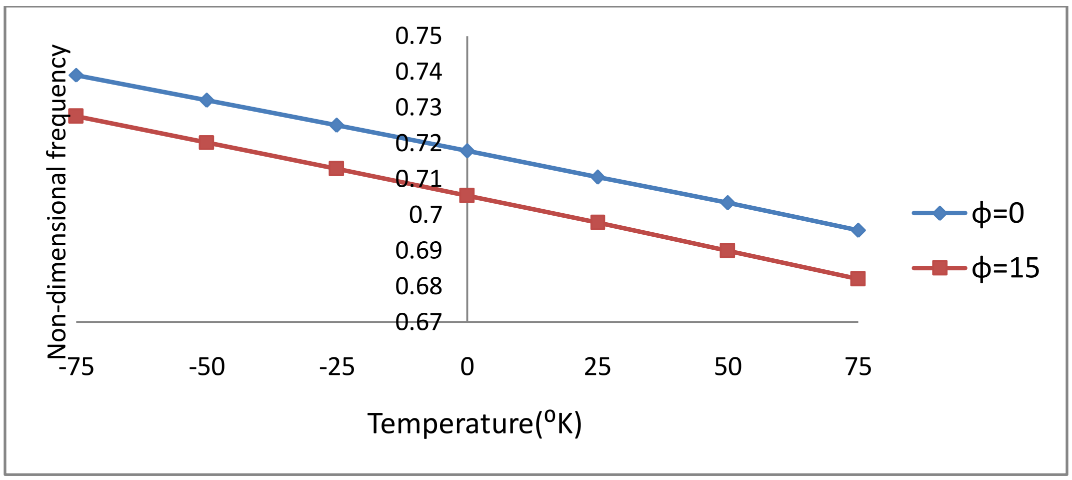


Fig 23 Effect of temperature on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/90/0)s for different angle of twist (a/b=3, b/t=20)

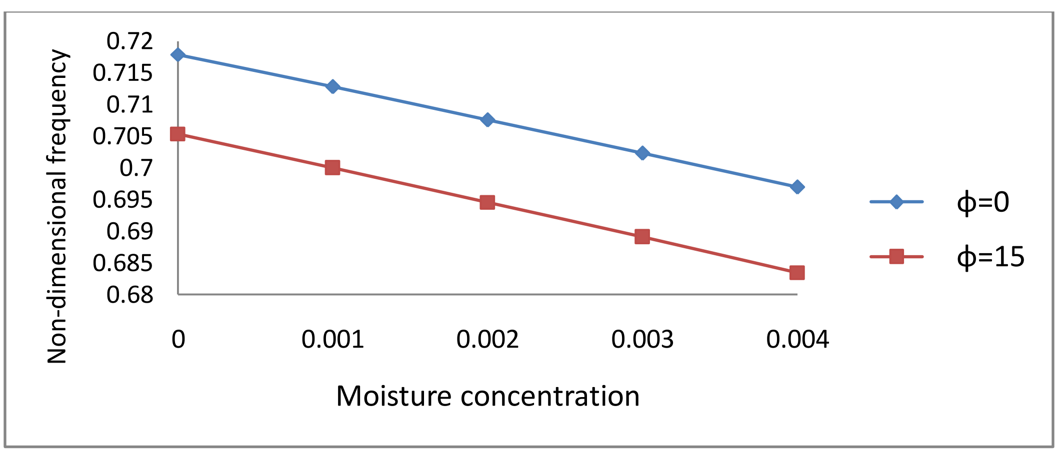


Fig 24 Effect of moisture on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/90/0)s for different angles of twist (a/b=3, b/t=20)

4.3.3.5 Buckling results for plates

The variation of non dimensional critical load due to effect of temperature and moisture for laminated composite twisted cantilever cross ply and angle ply plates is presented here. The variation of non dimensional critical load due to effect of temperature for angle ply laminated pre twisted cantilever plates is shown in figure 25.It is shown that as the temperature increases there is a decrease in the non dimensional buckling load for different angle ply orientation. As the temperature increases the percentage decrease in non dimensional critical load for [0/0/0/0]s ply orientation is 10.59%, for [15/-15/15/-15]s ply orientation, it is 16.12% and for [30/-30/-30/30]s ply orientation, it is about 41.83%. This result indicates that with increase in temperature the non dimensional buckling load decreases for angle ply plates.

The effect of moisture on non dimensional critical load for angle ply laminated pre twisted cantilever plates are presented in figure 26.As the moisture concentration increases the percentage decrease in non-dimensional frequency is 19.37% for [0/0/0/0]s ply orientation, 26.67% for [15/-15/-15/15]s ply orientation and 59.24% for[30/-30/-30/30]s ply orientation. This shows as the ply orientation increases the non dimensional buckling load decreases with increase in moisture for angle ply plates. The graph shows that percentage decrease for [30/-30/-30/30]s angle ply laminates is more than for [0/0/0/0]s and [15/-15/-15/15]s.

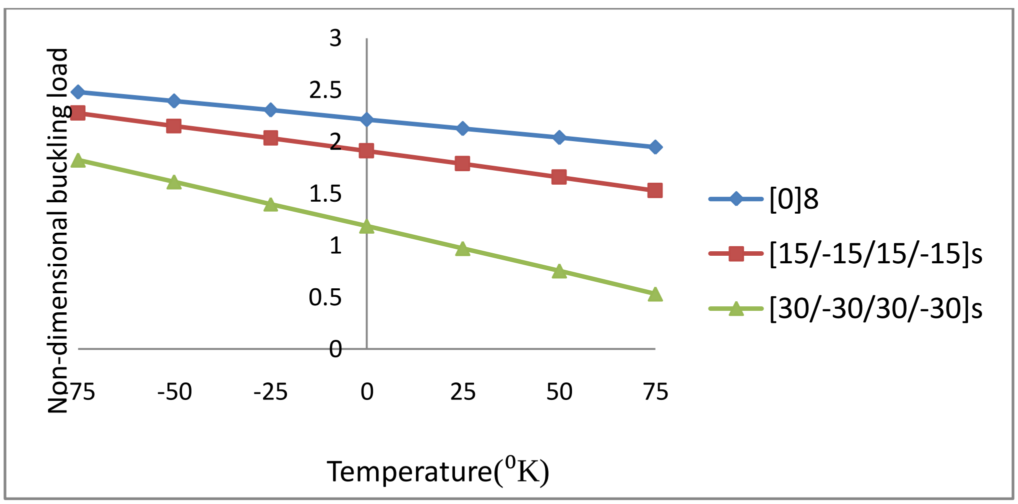


Fig 25:Effect of temperature on non-dimensional critical load for ϕ =30^0 and for angle-ply laminated pre twisted cantilever plates (a /b=1, b/t=20)

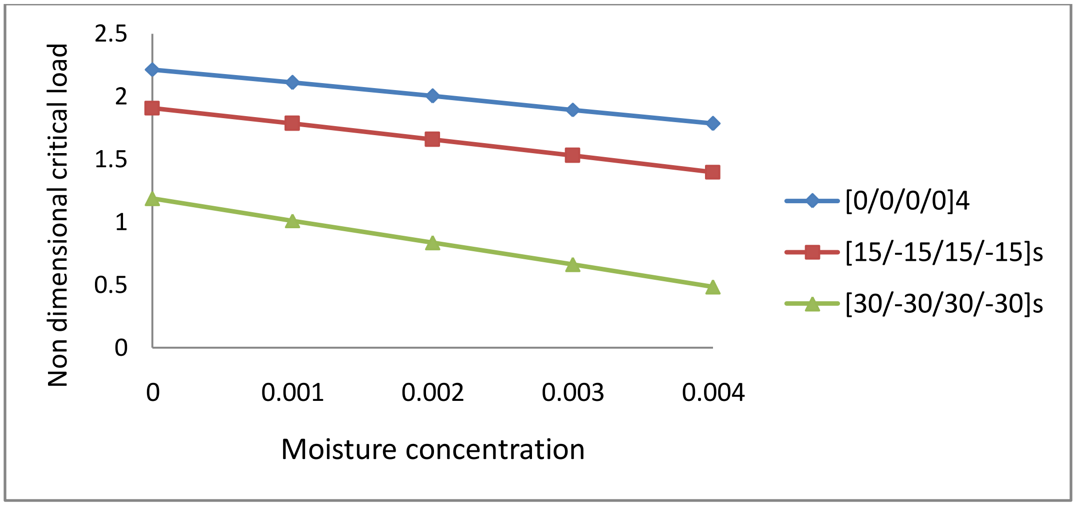


Fig 26:Effect of moisture on non-dimensional critical load for ϕ =30^0 and for angle-ply laminated pre twisted cantilever plates (a /b=1, b/t=20)

The variation of critical load with increase in temperature for cross ply laminated pre twisted cantilever plates is presented in figure 27. With increase in temperature from 225 K to 300 K the percentage decrease in non dimensional buckling load for un-symmetric and symmetric cross ply laminates are 41.31 % and 45.77 % respectively. When temperature increases from 300 K to 375 K the percentage decrease in non dimensional buckling load for un-symmetric and symmetric cross ply laminates are 82.62 % and 42.94% respectively This result shows that anti symmetric cross ply laminates are more unstable towards increase in temperature.

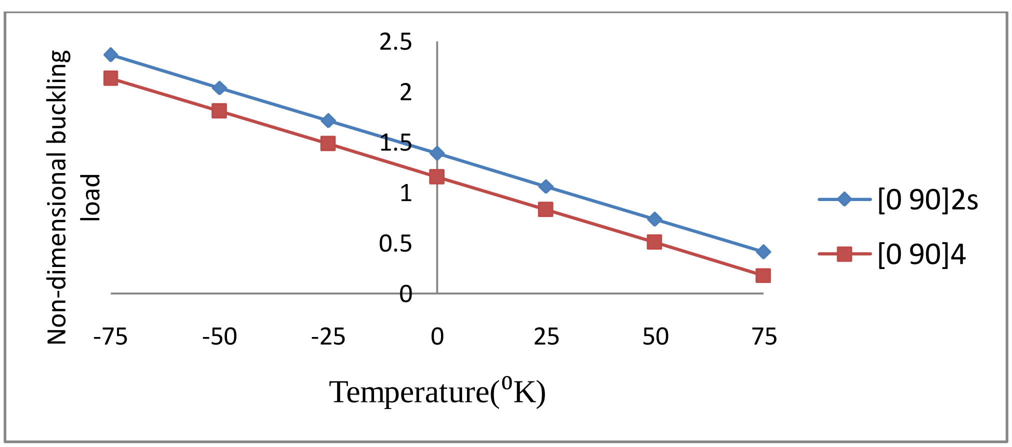


Fig27: Effect of temperature on non-dimensional critical load for ϕ =30^0 and for cross-ply laminated pre twisted cantilever plates (a /b=1, b/t=20)

The variation of critical load with increase in moisture for cross ply laminated pre twisted cantilever plates is presented in figure 28. With increase in moisture concentration the percentage decrease in non dimensional buckling load for symmetric and un-symmetric cross ply laminates are 66.41% and 79.66% respectively. This shows anti symmetric cross ply laminates are more unstable towards increase in moisture concentration than symmetric cross-ply laminates

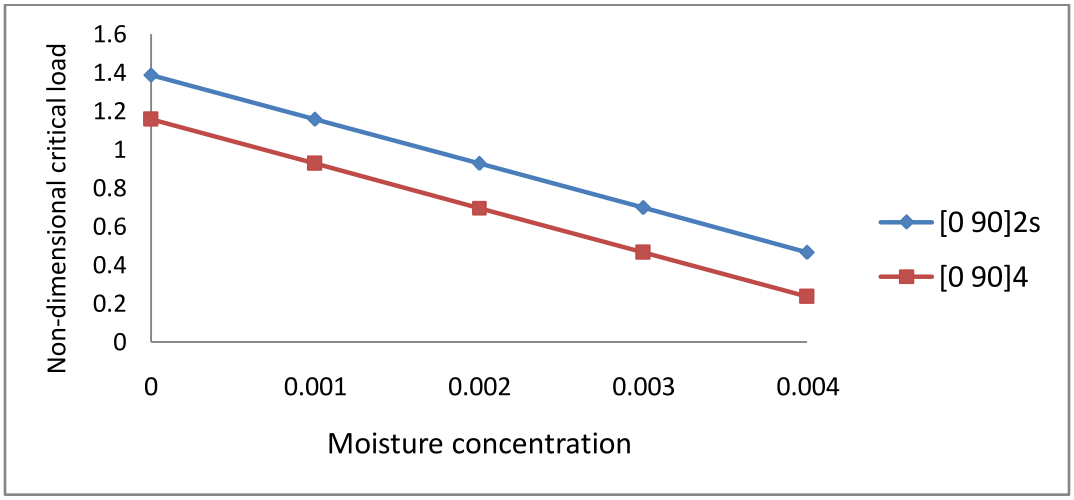


Fig 28: Effect of moisture on non-dimensional critical load for $\phi=30^0$ and for cross-ply laminated pre twisted cantilever plates (a /b=1, b/t=20)

4.3.3.6 Effect of angle of twists on buckling of pre twisted cantilever plates

The variation of non dimensional buckling load with increase in temperature for angle ply laminate for different angles of twists (ϕ) is shown in fig 26.As the temperature increases the percentage decrease in non dimensional critical load for twist angle 0^0 is about 65% and for twist angle 15^0 percentage decrease in non dimensional critical load is 65%. This result shows that the percentage decrease for angle ply laminate with increase in temperature is same. The decrease in non dimensional buckling load for twist angle 30^0 and 45^0 is about 70.99%, 19.94% and 85.17% respectively.

The variation of non dimensional buckling load with increase in moisture concentration for angle ply pre twisted cantilever plate for different angles of twists (ϕ) is shown in fig 27. The decrease in non dimensional buckling load for twist angle 0^0 with increase in moisture concentration from 0.1% to 0.4% is about 29.66%. This shows there is a sharp decrease in critical load for the plate when the temperature rises from 0.1% to 0.3%. The decrease in non dimensional critical load for twist angle 15^0, 30^0 and 45^0 while the moisture concentration increases from 0.1% to 0.4% is about 32.71%, 31.90% and 43.57% respectively.

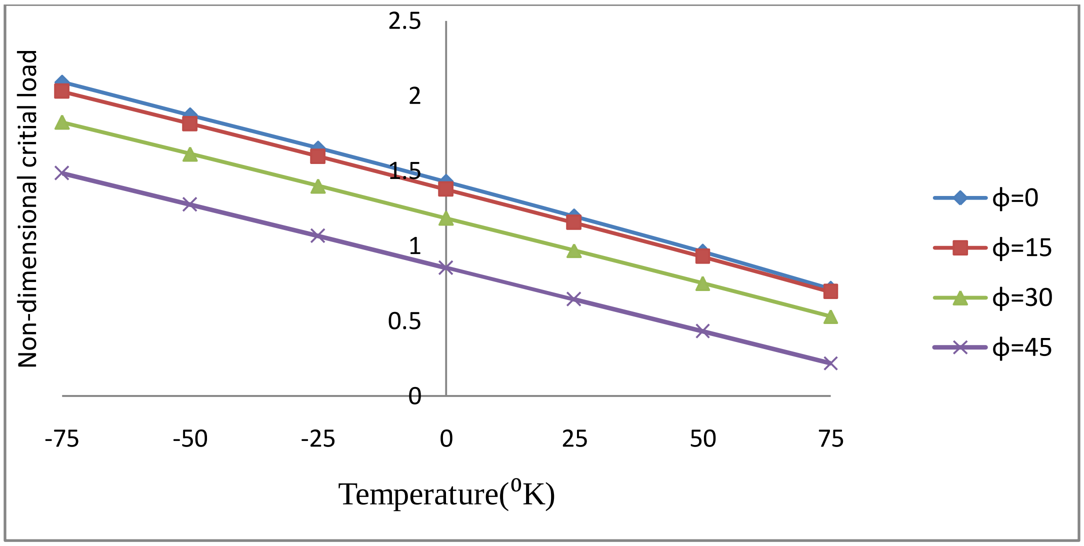


Fig 29:Effect of temperature on non-dimensional buckling load for angle ply pre twisted cantilever plate [30/-30/30/-30]s for different angles of twist (a/b=1, b/t=20)

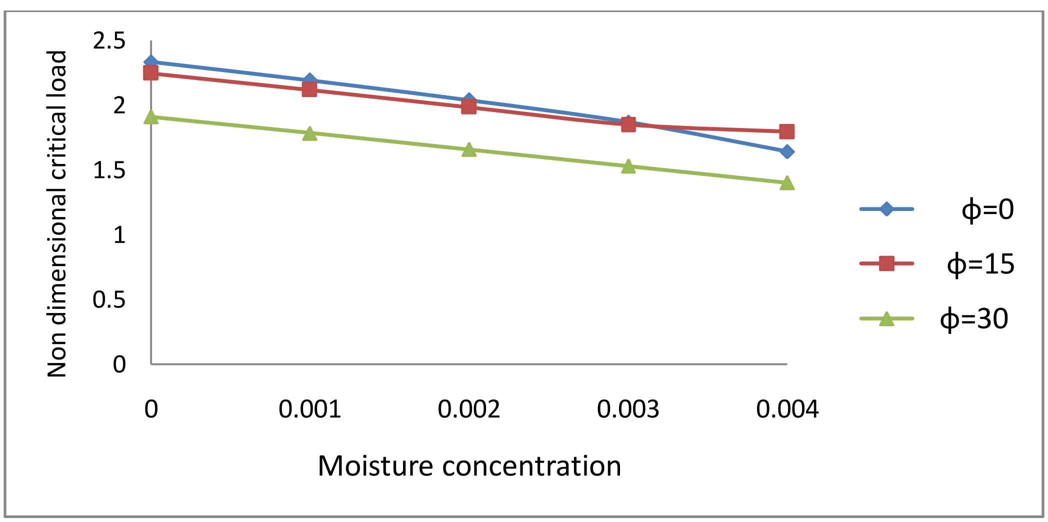


Fig 30:Effect of moisture on non-dimensional buckling load for laminated angle ply pre twisted cantilever plate [30/-30/30/-30]s for different angles of twist (a/b=1, b/t=20)

The variation of non dimensional buckling load with increase in temperature for symmetric cross ply laminate for different angles of twists (ϕ) shown in Fig 31. The decrease in non dimensional buckling load for twist angle 0^0, 15^0, 30^0 and 45^0 while the temperature increases from 225 K to 300 K is about 35.97.28%, 37.18%, 41.3 % and 49.9 % respectively. The decrease in non dimensional buckling load for twist angle 0^0, 15^0, 30^0 and 45^0 while the temperature increases from 300 K to 375 K is about 78.28%, 74.37%, 82.61% and 99.81% respectively. This result shows that non-dimensional buckling load decreases with increase in temperature.

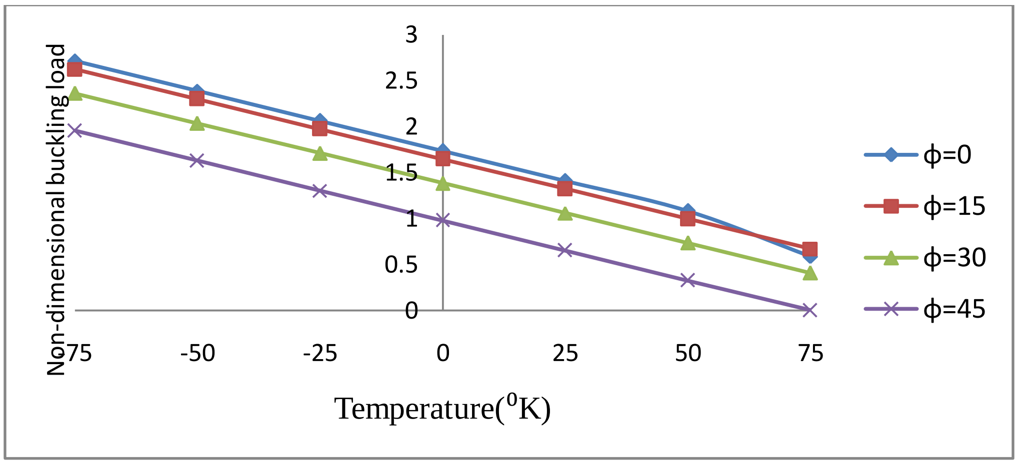


Fig 31:Effect of temperature on non-dimensional buckling load for laminated cross ply pre twisted cantilever plate (0/90/90/0)s for different angles of twist (a/b=1, b/t=20)

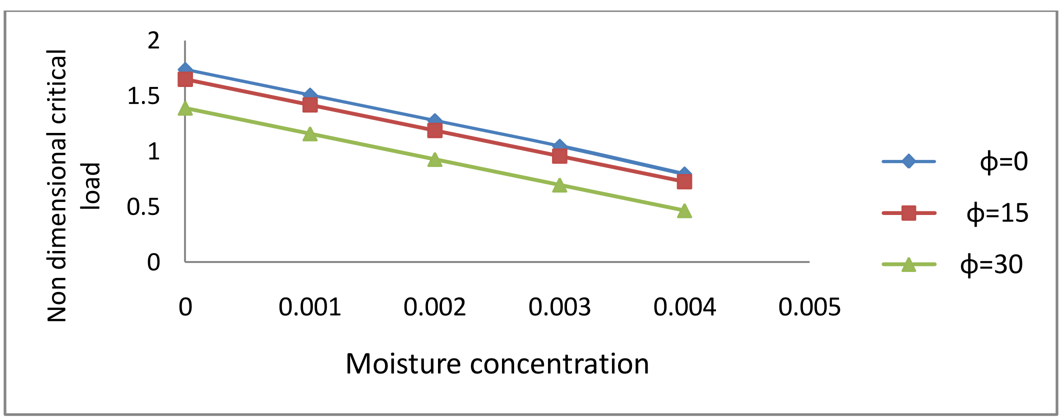


Fig 32:Effect of moisture on non-dimensional buckling load for laminated cross ply pre twisted cantilever plate (0/90/90/0)s for different angles of twist (a/b=1, b/t=20)

In Figure 32, the variation of non dimensional buckling load with increase in moisture concentration for cross ply laminate for different angles of twists (ϕ). The decrease in non dimensional buckling load for twist angle 0^0 while the moisture concentration increases from 0.1% to 0.4% is about 57.83% and for angle of twist $15^0$and 30^0, while the moisture concentration increases from 0.1% to 0.4% is about 54.10%, and 55.85% respectively.

The variation of non dimensional buckling load with increase in temperature from 225K to 375K for cross ply pre twisted cantilever panel (0/90/0/90)s for different angles of twists (ϕ) is shown in Fig 33. The decrease in non dimensional buckling load for twist angle 0^0 is about 15.22%. The decrease in non dimensional critical load for twist angle 15^0 and 30^0 while the temperature increases is about 15.99 % and 19.22 % respectively. This shows that decrease in critical load for angle of twist 0^0 and for angle of twist 15^0 are almost same.

The variation of non dimensional buckling load with increase in moisture concentration for cross-ply (0/90/0/90) for different angles of twists (ϕ) is shown in fig34. The decrease in non dimensional buckling load for twist angle 0^0 and 15^0, while the moisture concentration increases from 0.1% to 0.4% is about % 67.01% and 67.03%. The decrease in non dimensional critical load for twist angle 30^0 while the moisture concentration increases from 0.1% to 0.4%is about 58.44% respectively. This indicate that the non dimensional buckling load is same for angle of twist $0^0$and 15.

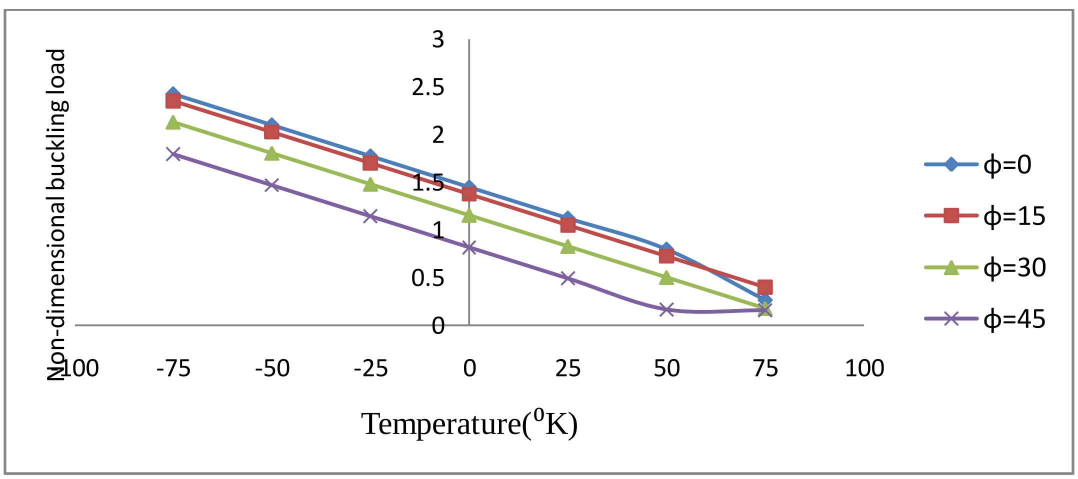


Fig 33:Effect of temperature on non-dimensional buckling load for laminated cross ply pretwisted cantilever plate (0/90/0/90)s for different angles of twist (a/b=1, b/t=20)

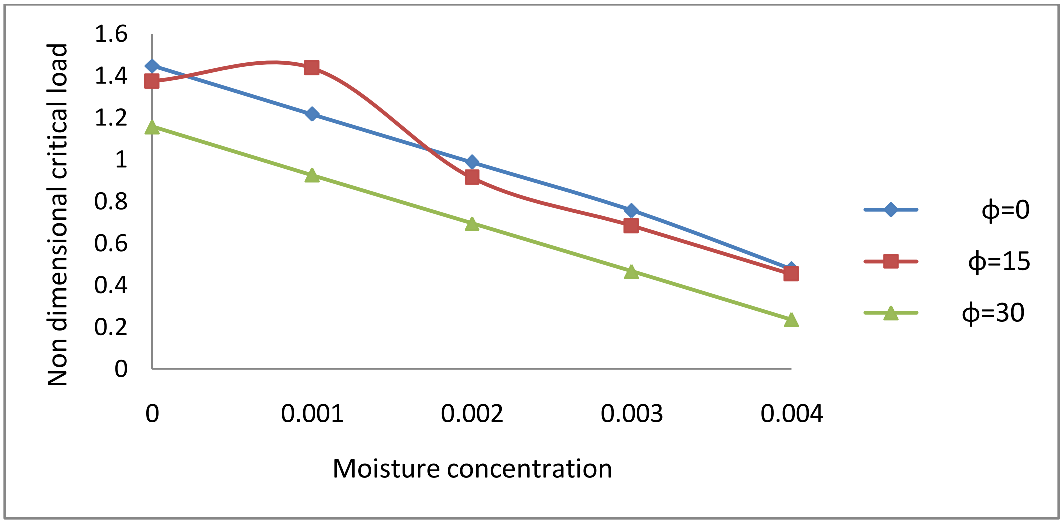


Fig 34:Effect of moisture on non-dimensional buckling load for laminated cross ply pre twisted cantilever plate (0/90/0/90)s for different angles of twist (a/b=1, b/t=20)

4.3.3.6 Effect of change in aspect ratio on buckling of composite twisted plates a/b=3

The variation of non dimensional critical load due to effect of temperature and moisture for laminated composite twisted cantilever cross ply and angle ply plates is presented here. The variation of non dimensional critical load due to effect of temperature for angle ply laminated pre twisted cantilever plates is shown in figure 35.It is shown that as the temperature increases from 225K to 375K there is a decrease in the non dimensional buckling load for different angle ply orientation. As the temperature increases the percentage decrease in non dimensional critical load for [0/0/0/0] s ply orientation is 3.01%, for [15/-15/15/-15]s ply orientation, it is 4.71% and for [30/-30/-30/30]s ply orientation, it is about 11.63%. This result indicates that with increase in temperature the non dimensional buckling load decreases for angle ply plates.

The effect of moisture on non dimensional critical load for angle ply laminated pre twisted cantilever plates are presented in figure 26.As the moisture concentration increases the percentage decrease in non-dimensional frequency is 2.48% for [0/0/0/0]s ply orientation, 3.29%

for [15/-15/-15/15]s ply orientation and 6.68% for[30/-30/-30/30]s ply orientation. This shows as the ply orientation increases the non dimensional buckling load decreases with increase in moisture for angle ply plates. The graph shows that percentage decrease for [30/-30/-30/30]s angle ply laminates is more than for [0/0/0/0]s and [15/-15/-15/15]s.

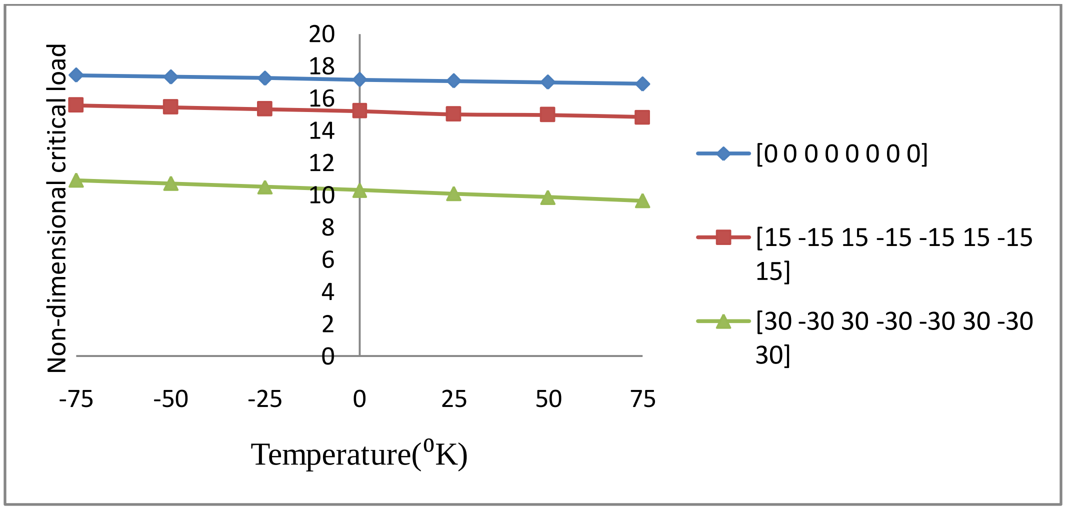


Fig 35:Effect of temperature on non-dimensional critical load for ϕ =30^0 and for angle-ply laminated pre twisted cantilever plates (a /b=3, b/t=20)

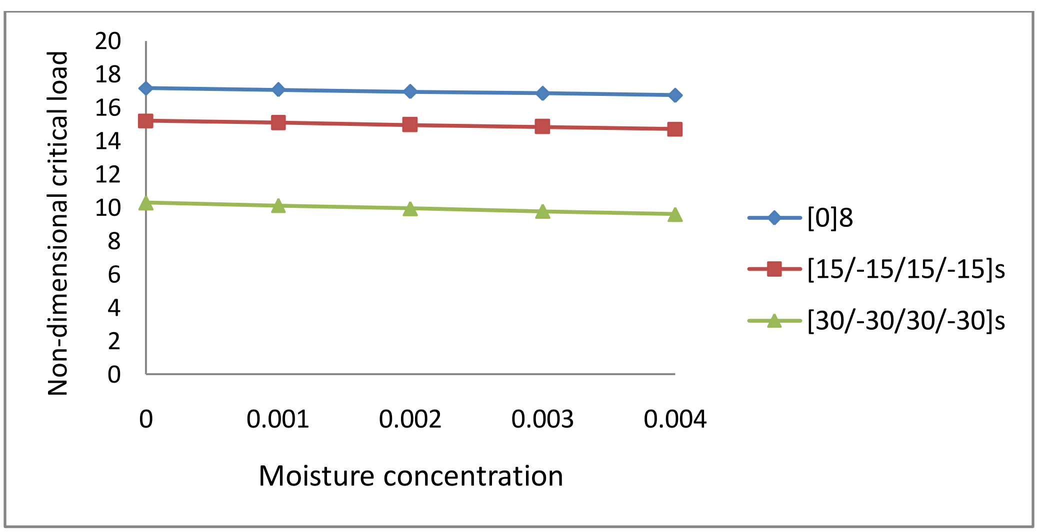


Fig 36:Effect of moisture on non-dimensional critical load for ϕ =30^0 and for angle-ply laminated pre twisted cantilever plates (a /b=3, b/t=20)

The variation of critical load with increase in temperature for cross ply laminated pre twisted cantilever plates is presented in figure 37. With increase in temperature the percentage decrease in non dimensional buckling load for symmetric and anti symmetric cross ply laminates are 16.33% and 19.22% respectively. This shows anti symmetric cross ply laminates are more vulnerable towards increase in temperature.

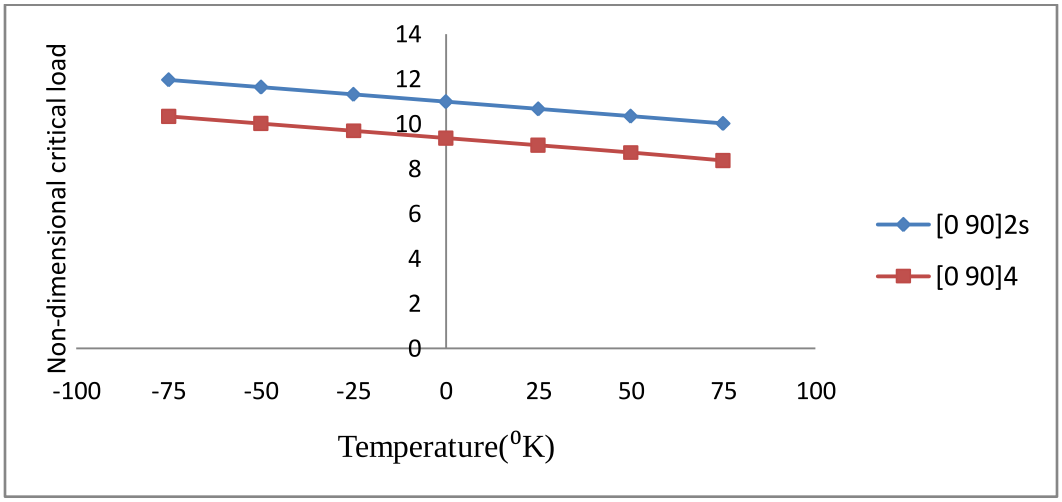


Fig 37:Effect of temperature on non-dimensional critical load for ϕ =30^0 and for cross-ply laminated pre twisted cantilever plates (a /b=3, b/t=20)

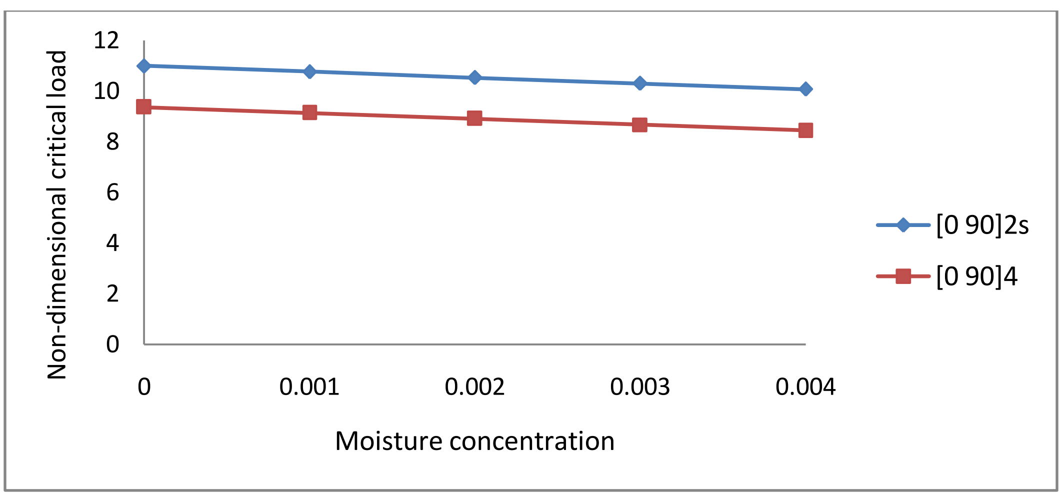


Fig 38:Effect of moisture on non-dimensional critical load for ϕ=30^0 and for cross-ply laminated pre twisted cantilever plates (a /b=3, b/t=20)

The variation of critical load with increase in moisture for cross ply laminated pre twisted cantilever plates is presented in figure 38. With increase in moisture concentration the percentage decrease in non dimensional buckling load for symmetric and anti symmetric cross ply laminates are 9.83% and 8.38 % respectively. This shows anti symmetric cross ply laminates are more vulnerable towards increase in moisture concentration.

4.3.3.5.7 Effect of angle of twists on buckling of pre twisted cantilever plates

The variation of non dimensional buckling load with increase in temperature for angle ply laminate for different angles of twists (ϕ) is shown in fig 39.As the temperature increases from 275 K to 300 K the percentage decrease in non dimensional critical load for twist angle 0^0 is about 4.93% and for twist angle 15^0 percentage decrease in non dimensional critical load is 5.02%. This result shows that the percentage decrease for angle ply laminate with increase in temperature is same. The decrease in non dimensional buckling load for twist angle 30^0 is 5.80%. from temperature 225 K to 300 K. As the temperature increases from 300 K to 375 K the percentage decrease in non dimensional critical load for twist angle 0^0 is about 9.88% and for twist angle 15^0 percentage decrease in non dimensional critical load is 10.12%. This result shows that the percentage decrease for angle ply laminate with increase in temperature is same. The decrease in non dimensional buckling load for twist angle 30^0 is 5.80%. from temperature 300 K to 375 K.

The variation of non dimensional buckling load with increase in moisture concentration for angle ply pre twisted cantilever plate for different angles of twists (ϕ) is shown in fig 40. The decrease in non dimensional buckling load for twist angle 0^0 with increase in moisture concentration from o.1%to 0.4% is about 5.60 %. This shows there is a sudden decrease in critical load for the plate when the temperature rises from 0.1% to 0.4%. The decrease in non dimensional critical load for twist angle 15^0, 30^0 and 45^0 while the moisture concentration increases from 0.1% to 0.4% is about 5.76%, and 6.68 % respectively.

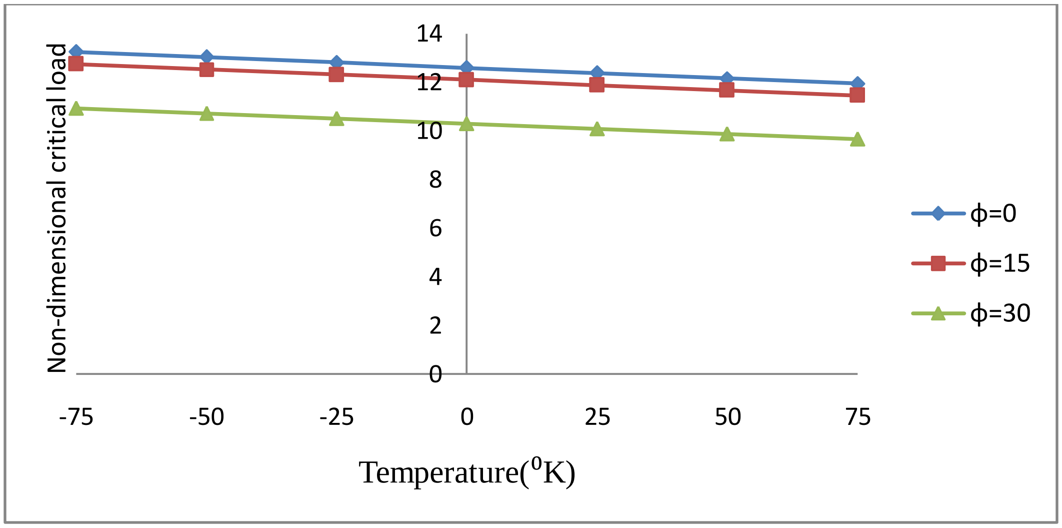


Fig 39:Effect of temperature on non-dimensional buckling load for laminated angle ply pretwisted cantilever plate [30/-30/30/-30]s for different angles of twist (a/b=3, b/t=20)

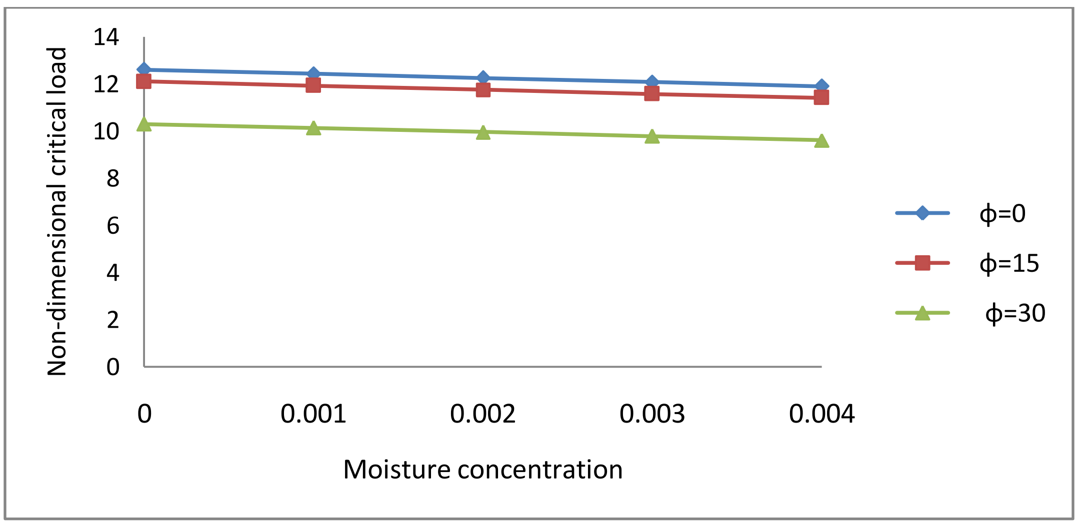


Fig 40:Effect of moisture on non-dimensional buckling load for laminated angle ply pre twisted cantilever plate (30/-30/30/-30)s for different angles of twist (a/b=3, b/t=20)

The variation of non dimensional buckling load with increase in temperature for cross ply pre twisted cantilever panel (0/90/90/0)s for different angles of twists (ϕ) is shown in Fig41. The decrease in non dimensional buckling load for twist angle 0^0 is about 13.07%. The decrease in non dimensional critical load for twist angle 15^0 and 30^0 while the temperature increases is about13.77% and 16.33 % respectively. This shows that decrease in critical load for angle of twist 0^0 and for angle of twist 15^0 are almost same.

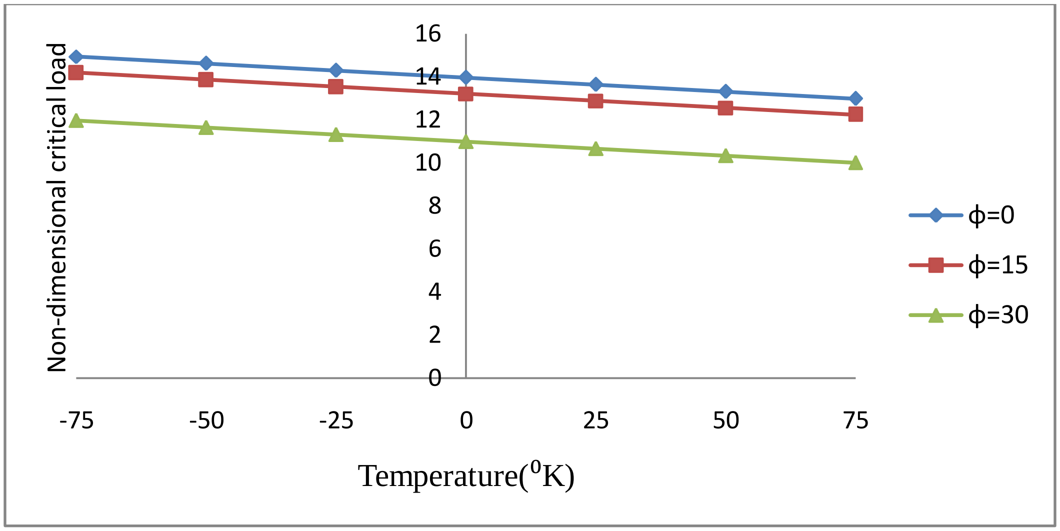


Fig 41:Effect of temperature on non-dimensional buckling load for laminated cros-ply pre twisted cantilever plate (0/90/90/0)s for different angles of twist (a/b=3, b/t=20)

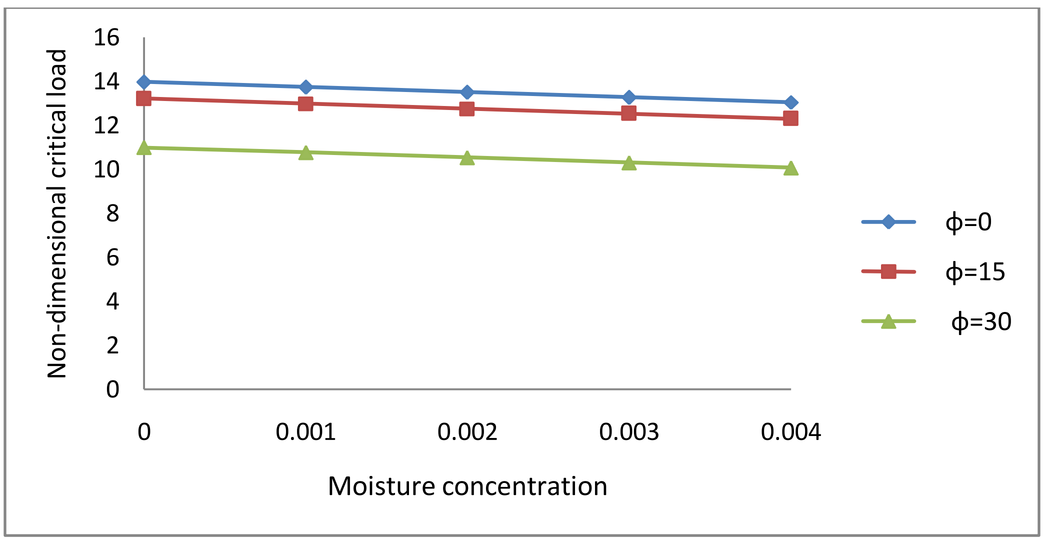


Fig 42:Effect of moisture on non-dimensional buckling load for laminated cross ply pre twisted cantilever plate (0/90/90/0)s for different angles of twist (a/b=3, b/t=20)

The variation of non dimensional buckling load with increase in moisture concentration for cross-ply (0/90/90/0) for different angles of twists (ϕ) is shown in fig42. The decrease in non dimensional buckling load for twist angle 0^0and15^0, while the moisture concentration increases from 0.1% to 0.4% is about % 6.60% and 6.98%. The decrease in non dimensional critical load for twist angle 30^0 while the moisture concentration increases from 0.1% to 0.4%is about 8.38% respectively. This indicate that the non dimensional buckling load is same for angle of twist $0^0$and 15^0.

The variation of non dimensional buckling load with increase in temperature for cross ply pre twisted cantilever panel (0/90/0/90)s for different angles of twists (ϕ) is shown in Fig43. The decrease in non dimensional buckling load for twist angle 0^0 is about 15.22%. The decrease in non dimensional critical load for twist angle 15^0 and 30^0 while the temperature increases is about 15.99% and 19.22 % respectively. This shows that decrease in critical load for angle of twist 0^0 and for angle of twist 15^0 are almost same.

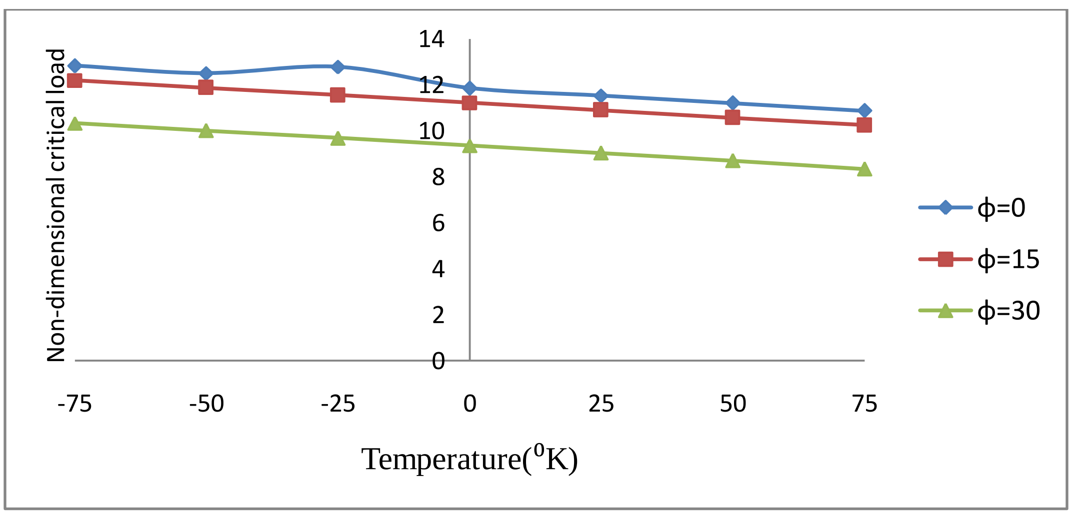


Fig 43:Effect of temperature on non-dimensional buckling load for laminated cross ply pretwisted cantilever plate (0/90/0/90)s for different angles of twist (a/b=3, b/t=20)

The variation of non dimensional buckling load with increase in moisture concentration for cross-ply (0/90/0/90) for different angles of twists (ϕ) is shown in fig34. The decrease in non dimensional buckling load for twist angle 0^0and15^0, while the moisture concentration increases from 0.1% to 0.4% is about % 7.77% and 8.20%. The decrease in non dimensional critical load for twist angle 30^0 while the moisture concentration increases from 0.1% to 0.4%is about 9.83% respectively. This indicate that the non dimensional buckling load is same for angle of twist $0^0$and 15^0

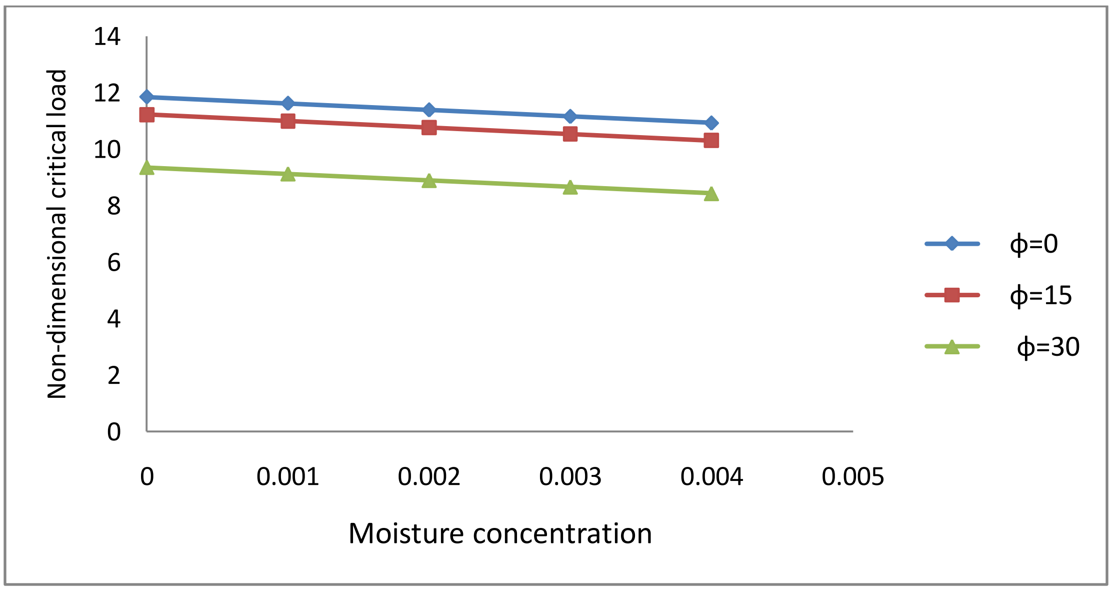


Fig 44:Effect of moisture on non-dimensional buckling load for laminated cross ply pre twisted cantilever plate (0/90/0/90)s for different angles of twist (a/b=1, b/t=20)

CHAPTER 5

CONCLUSION

In the present study the conventional finite element formulation is modified to study the free vibration and stability of laminated composite twisted cantilever panels subjected to hygrothermal conditions. An eight noded isoparametric shell element is used for this analysis with five degrees of freedom at each node. The formulation and program developed are general in nature and can handle non-uniform distributions of moisture and temperature. The numerical results for free vibration and buckling are presented and discussed above. The broad conclusion that can be made from this analysis is summarized below.

- The fundamental frequency decrease with increase in temperature and moisture concentration.
- For symmetric angle ply twisted cantilever panels as the temperature and moisture increases the frequency decreases with increase in ply orientation.
- It is found that as the angle of twist increases for a particular cross-ply orientation, the frequency parameter decreases. This is true for both the symmetric as well as antisymmetric cross-ply stacking sequences.
- For a particular ply orientation, the non dimensional frequency is less decreasing as the aspect ratio increases.
- The effect of temperature and moisture is much more in un-twisted plate than twisted plate.
- For twisted cantilever cross-ply laminates the percentage decrease in frequency is more for anti symmetric laminates than symmetric laminates.
- The critical load decrease with increase in temperature and moisture concentration.
- The buckling load of angle ply and cross-ply twisted cantilever panel decreases as the temperature and moisture concentration increases.
- The buckling loads tend to decrease with increase of lamination angle from 0° to 30^0 for untwisted and twisted plates.

- For all ply orientation, the non dimensional critical load is less decreasing as the aspect ratio increases.

From the above study, it is concluded that the vibration and buckling behaviour of the laminated composite twisted cantilever plates and shells are deeply influenced by the geometry, lamination parameter, angle of twist and hygrothermal condition. Therefore, the designer has to be cautious while dealing with structures subjected to hygrothermal loading. So, this can be used to the advantage of tailoring during design of composite twisted cantilever panels.

Scope for future work

The possible extensions to the present study are presented below:

- The present investigation can be extended to dynamic stability studies of cantilever twisted panels subjected to hygrothermal loading.
- The twisted panels in this study are of uniform thickness and width. Hence the effect of varying thickness and width may also be incorporated in this study.
- Material non linearity can be taken into account for further studies of twisted cantilever panels.
- There is also a scope to study the vibration and buckling of twisted cantilever panels subjected to hygrothermal loading by experimental method.

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